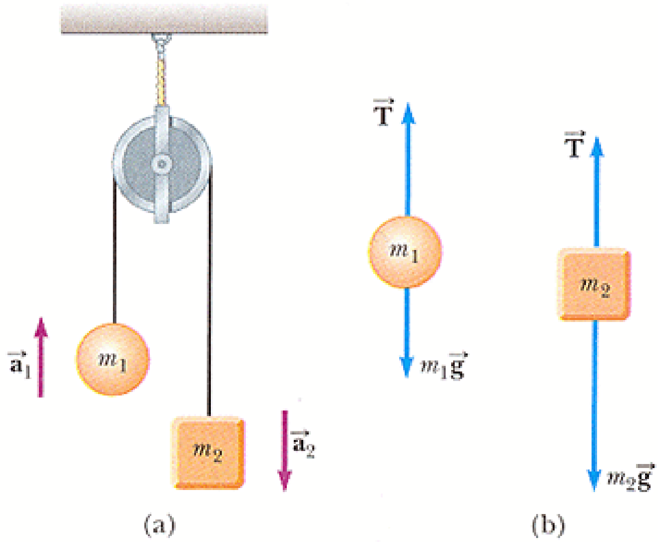
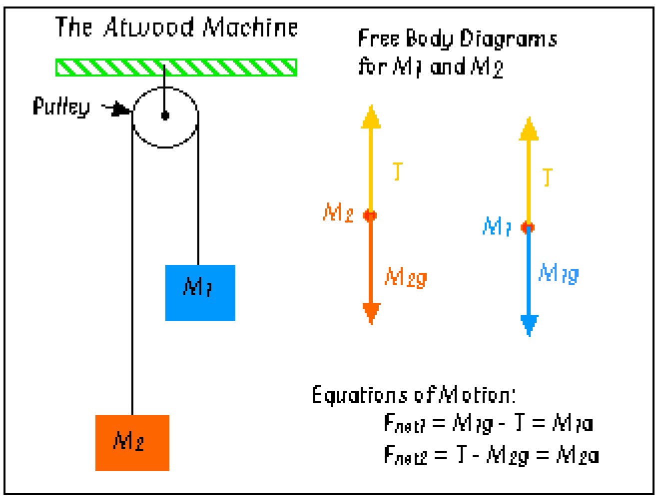
Atwood's Machine Examples

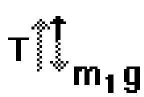




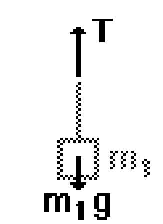
Frictionless case, neglecting pulley mass

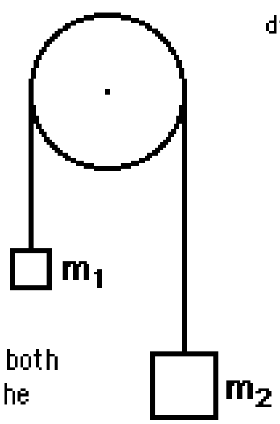
Equation of motion for m_1

$$F_{net} = T - m_1 g = m_1 a$$

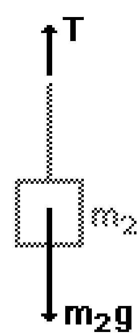


Free-body diagram for m_1



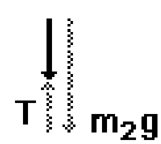


Free-body diagram for m_2



Equation of motion for m_2

$$F_{net} = m_2 g - T = m_2 a$$



For this case the tension T is the same on both sides of the pulley. The acceleration a is the same for both. Solving for T gives:

$$T = m_1 g + m_1 a$$

Substituting T in the equation for m_2 gives

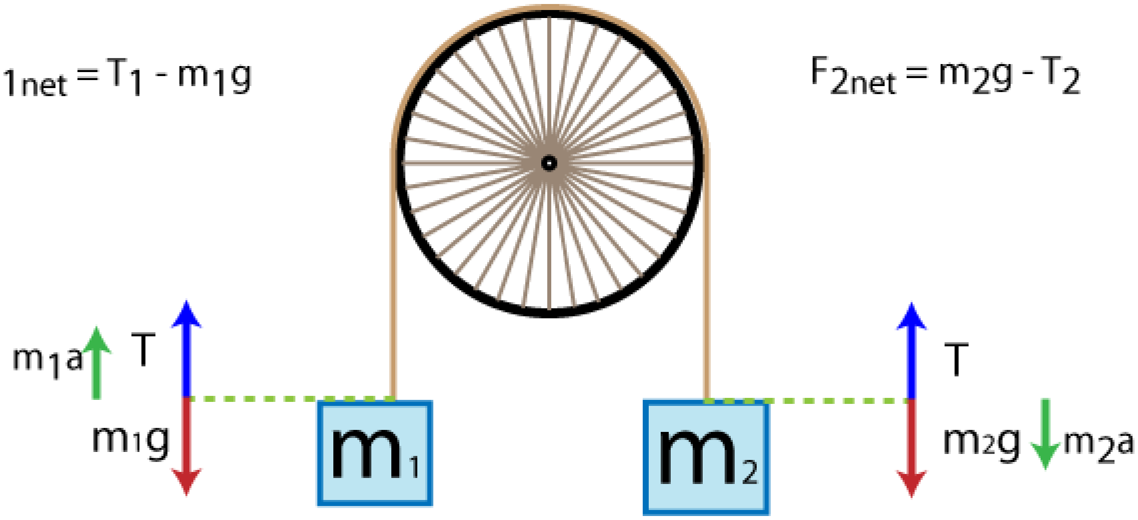
$$m_2 g - m_1 g - m_1 a = m_2 a$$

↓ a = acceleration

The system equation of motion can be written

$$(m_2 - m_1)g = (m_1 + m_2)a \quad \text{or} \quad a = \frac{(m_2 - m_1)g}{(m_1 + m_2)}$$

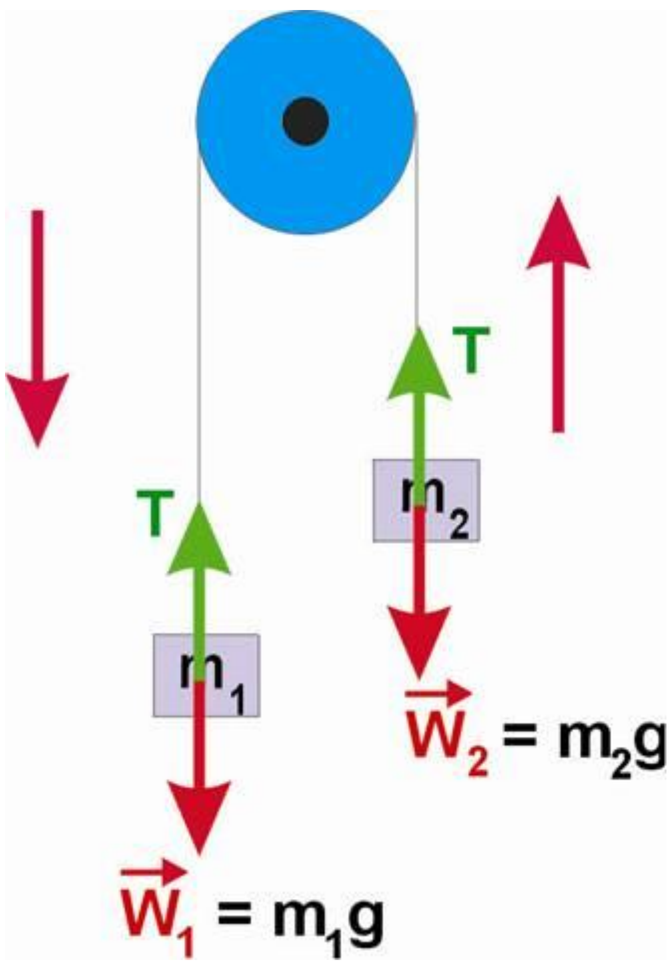
$$F_{1net} = T_1 - m_1 g \qquad F_{2net} = m_2 g - T_2$$

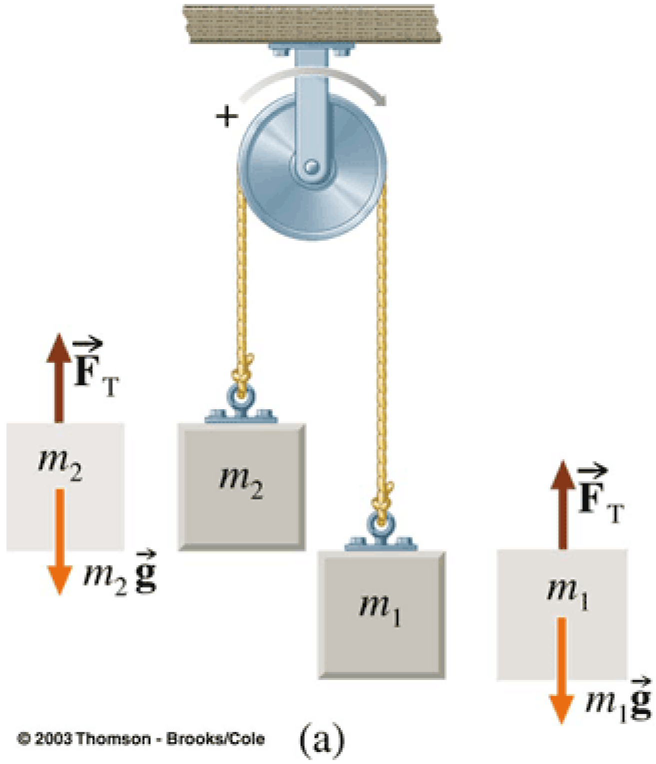


$$T_1 = T_2$$
$$T_2 = m_2 g - F_{2net}$$
$$F_{1net} = m_1 a$$
$$F_{2net} = m_2 a$$

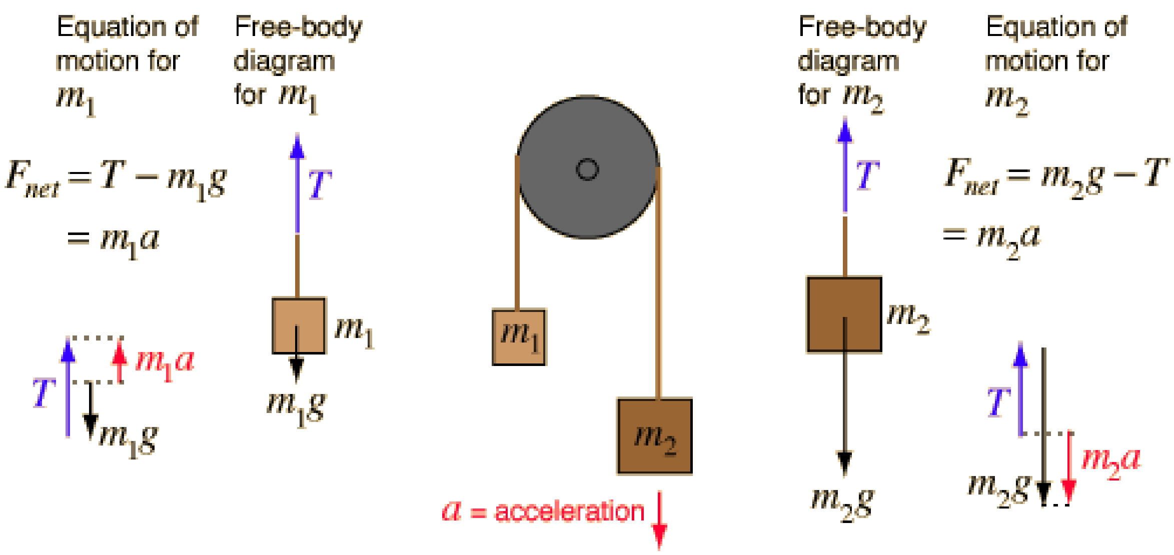
$$m_1 a = m_2 g - m_2 a - m_1 g$$

$$\boxed{(m_1 + m_2)a = (m_2 - m_1)g}$$





Frictionless case, neglecting pulley mass



For this idealized case the tension T is the same on both sides of the pulley. The acceleration a is the same for both masses. Solving for T gives:

$$T = m_1 g + m_1 a$$

Substituting T into the equation for m_2 gives

$$m_2 g - m_1 g - m_1 a = m_2 a$$

The equation of motion for the two-mass system is then:

$$(m_2 - m_1)g = (m_1 + m_2)a \quad \text{or} \quad a = \frac{(m_2 - m_1)g}{(m_1 + m_2)}$$