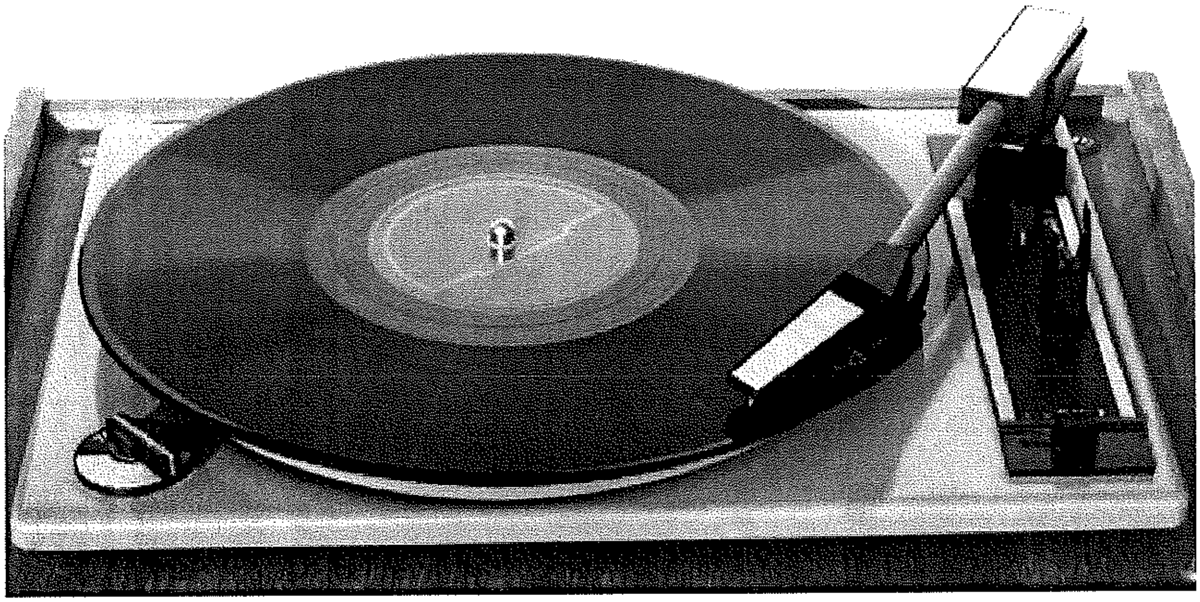
Question:
A car travels at constant speed v in a circle of radius r. Which of the following statements is true?

a. The car's angular velocity is rv.
b. The car has a non-zero tangential acceleration.
c. The car has a constant magnitude of centripetal acceleration.
d. The car travels at constant speed and has *no* acceleration.
e. none of the statements above is true.

Answer:
The correct answer is *c*. The car's centripetal acceleration is $a=v^2/r$, and although the direction of this acceleration is directed toward the center of the circle and therefore keeps changing as the car moves, the magnitude of that acceleration is constant.



Question:
A turntable makes a record rotate at 33.3 revolutions per minute. What is the rotational speed of a point on the outer edge of the record?

a. 33.3 revolutions per minute
b. Less than 33.3 revolutions per minute
c. More than 33.3 revolutions per minute
d. The question can't be answered without knowing the radius of the record
e. None of the above

Answer:
The correct answer is ***a***. The rotational speed of the record, 33.3 revolutions per minute, is the same for all points on the record. The *linear speed*, or *tangential speed*, of each point on the record does depend on the radius. The linear speed v is proportional to the radius of that point—the distance between the axis of rotation and the location of the point.

$$v \propto r\omega$$

Question:

The tire of a large truck has a diameter of 1.00m, and rolls along a horizontal surface without slipping. A point *P* located at the perimeter of the wheel has an angular speed of 6 radians/s relative to the axle of the wheel. What is the velocity of the truck?

a. 6 m/s
b. 12 m/s
c. 3 m/s
d. 6π m/s
e. 12π m/s

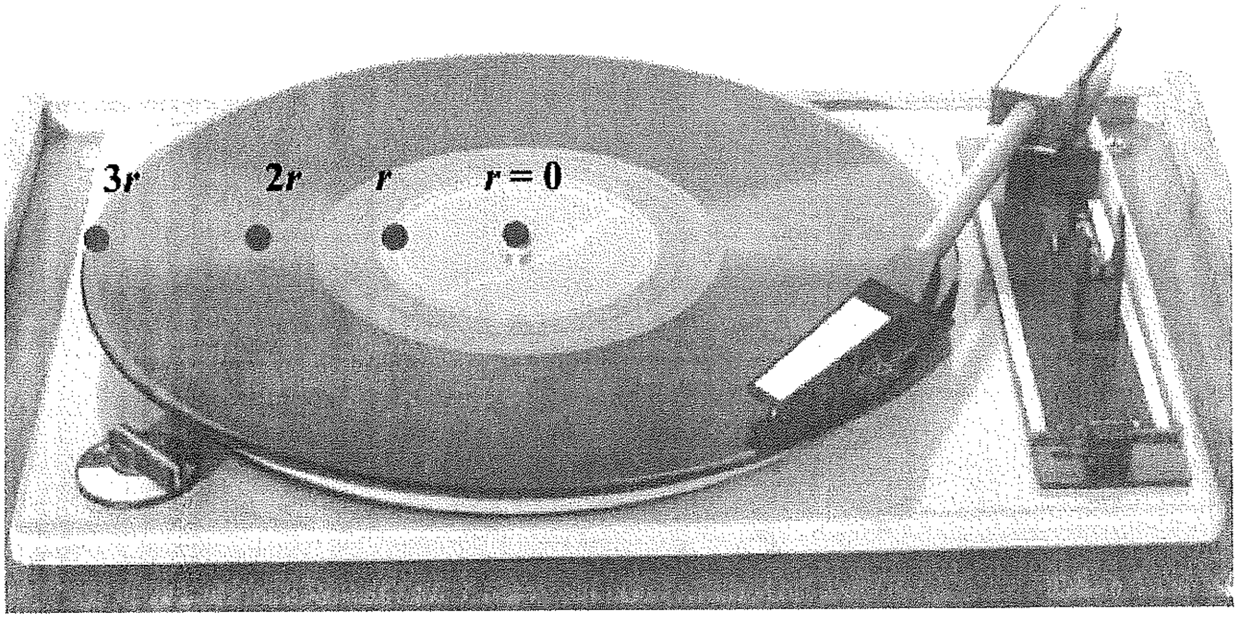
Answer:

The correct answer is *c*. The center of mass of the tire (and therefore the truck itself) moves according to $v_{cm} = r\omega$. Therefore:

$$v_{cm} = r\omega$$

$$v_{cm} = (\tfrac{1}{2}1.00m)(6\text{rad/s})$$

$$v_{cm} = 3m/s$$



Question:
The turntable shown above makes a record rotate at 33.3 revolutions per minute. If the linear speed of an ant located at radius r is v, what is the linear speed of a point on the outer edge of the record at radius $3r$?

a. also v

b. $\frac{1}{3}v$

c. $3v$

d. $9v$

e. $6v$

Answer:
The correct answer is *c*. While the rotational speed—typically given in rotations per minute, or RPM—is the same everywhere, the *linear speed* of a point depends on the location of that point relative to the axis of rotation.

The linear speed is directly proportional to the radius: $v \propto r\omega$. Here, because the edge of the record is at $3r$, the velocity v is three times the velocity of a point at r.

Question:

A long, thin, rod of with moment of inertia I=2 kg•m² is free to rotate about an axis passing through the midpoint of the rod. The rod begins rotating from rest at time t=0 seconds, accelerating constantly so that it has a rotational velocity of 4π rad/s after rotating through two complete revolutions. What is the rod's average angular acceleration during its motion?

a. $\frac{\pi}{4} rad/s^2$

b. $\frac{1}{4} rad/s^2$

c. $8\pi \ rad/s^2$

d. $4\pi \ rad/s^2$

e. $2\pi \ rad/s^2$

Answer:

The correct answer is *e*. The angular acceleration of the rod can be calculated using angular kinematics:

$$\omega_f^2 = \omega_i^2 + 2\alpha\theta$$

$$\alpha = \frac{\omega_f^2 - \omega_i^2}{2\theta}$$

$$\alpha = \frac{(4\pi \ rad/s)^2 - 0^2}{2\left(2rev \times \frac{2\pi \ rads}{1rev}\right)}$$

$$\alpha = \frac{16\pi^2}{8\pi} = 2\pi \ rad/s^2$$

Question:

A carousel—a horizontal rotating platform—of radius *r* is initially at rest, and then begins to accelerate constantly until it has reached an angular velocity ω after 2 complete revolutions. What is the angular acceleration of the carousel during this time?

a. $\dfrac{\omega^2}{8\pi}$

b. $\dfrac{\omega^2}{4\pi}$

c. $\dfrac{\omega}{4\pi}$

d. $\dfrac{\omega^2}{4\pi r}$

e. $\dfrac{\omega^2}{2\pi r}$

Answer:

The correct answer is ***a***. The angular acceleration of the carousel can be determined by using rotational kinematics:

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\alpha = \frac{\omega^2 - \omega_0^2}{2\theta}$$

$$\alpha = \frac{\omega^2}{2(2 \bullet 2\pi)} = \frac{\omega^2}{8\pi}$$

Question:

A turntable begins to rotate from rest with a constant angular acceleration. If a point on the edge of the turntable travels 9 radians during a 3.0 second time period, the angular acceleration of the turntable is

a. $2.0s^{-2}$
b. $3.0s^{-2}$
c. $4.0s^{-2}$
d. $6.0s^{-2}$
e. $9.0s^{-2}$

Answer:

The correct answer is *a*. The angular displacement of the turntable as it accelerates may be easily determined using rotational kinematics:

$$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

$$\alpha = \frac{2(\Delta\theta - \omega_i t)}{t^2} = \frac{2(9.0rad - 0)}{(3s)^2} = 2.0s^{-2}$$

Question:

Which of the following statements concerning rotational motion is(are) correct?

- **I.** **The radial linear acceleration a_{rad} is related to angular acceleration α.**
- **II.** **The linear acceleration a_{tan} is related to angular velocity ω.**
- **III.** **The rotational kinetic energy of a spinning object is determined by its rotational inertia and its angular velocity.**
- **IV.** **A rotating body that is spinning "up" will have its linear velocity vector v pointed in the same direction as its a_{rad} vector.**
- **V.** **A rotating extended body spinning with positive rotation in the plane of this page will have its angular velocity vector pointing into the page.**
- **VI.** **The direction of a spinning object's angular displacement vector ϑ will always point in the same direction as it angular velocity vector ω.**
- **VII.** **The direction of a spinning object's angular acceleration vector α will always point in the same direction as it angular velocity vector ω.**

Answers:

III, VI

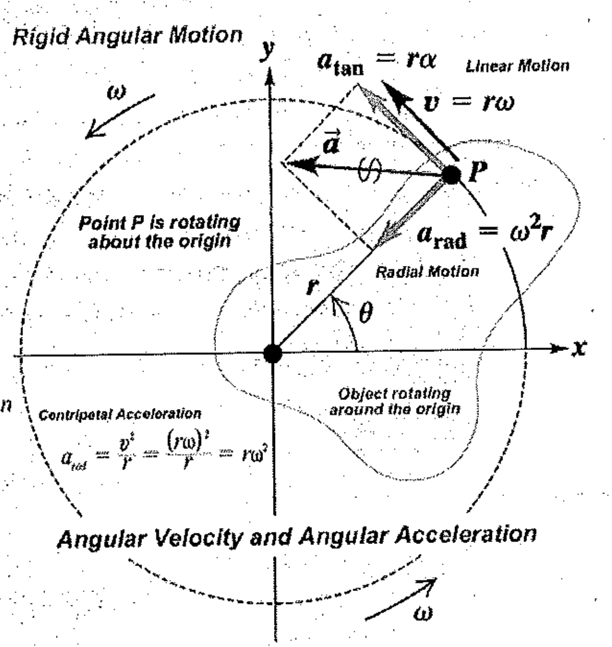
Derivation of Angular quantities from Linear quantities

Linear Velocity / *Angular Velocity*

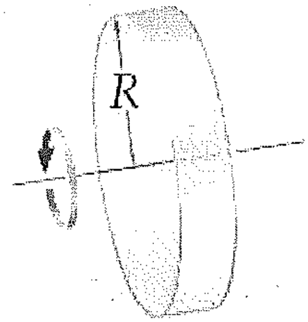
$$v = \frac{\Delta s}{\Delta t} = \frac{r\Delta\theta}{\Delta t} = r\omega$$

Linear Acceleration / *Angular Acceleration*

$$a = \frac{\Delta v}{\Delta t} = \frac{r\Delta\omega}{\Delta t} = r\alpha$$



Question:

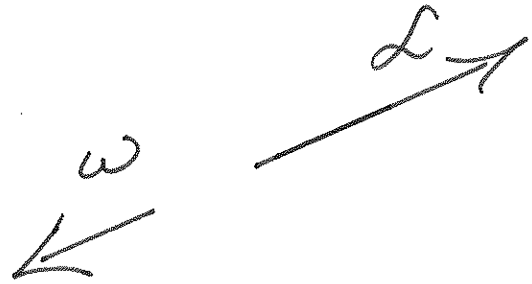


A solid disk attached to a shaft through its center is rotating as shown in the diagram above. Answer the following questions for the case of the disk slowing down.

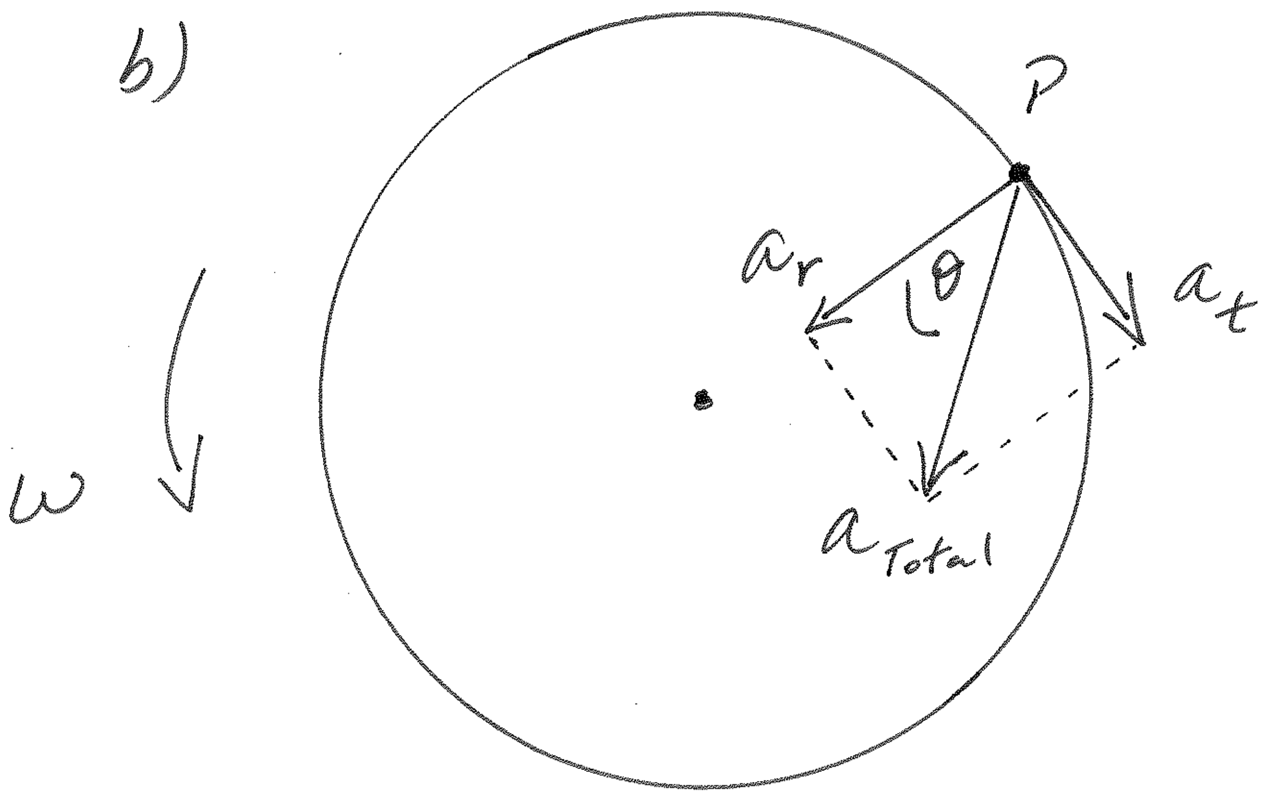
1. **Draw and label the *angular velocity* vector.**
2. **Draw and label the *angular acceleration* vector.**
3. **Draw and label the *radial, tangential,* and *total* acceleration vectors for a point on the rim of the disk.**

Answer:

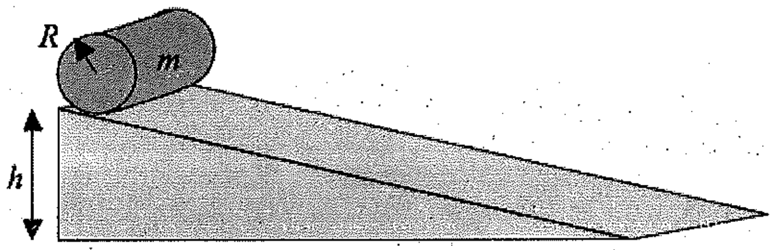
a)



b)



Question:



A cylinder of mass *m* and radius R has a moment of inertia of $\frac{1}{2}mR^2$. The cylinder is released from rest at a height *h* on a frictionless inclined plane, and slides down without rolling. What is the velocity *v* of the cylinder when it reaches the bottom of the incline?

a. $\sqrt{\frac{4}{3}gh}$

b. $\sqrt{\frac{1}{2}gh}$

c. $\sqrt{2gh}$

d. $\sqrt{\frac{3}{4}mgh}$

e. $\sqrt{gh}$

Answer:

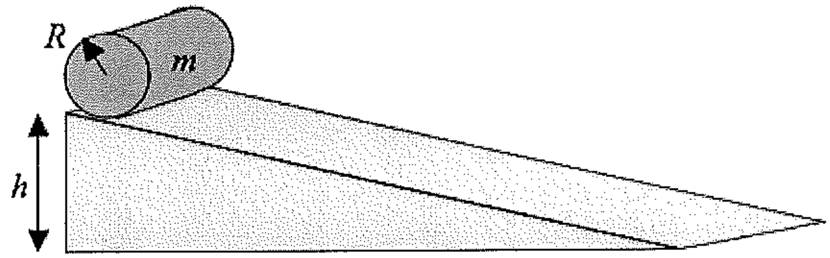
The correct answer is *c*. Because the cylinder is not rolling, this is a simple conservation of energy problem in which the gravitational potential energy at the top of the plane is converted to translational kinetic energy at the bottom.

$$U_g = K_{translational}$$

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

Question:



A cylinder of mass m and radius R has a moment of inertia of $\frac{1}{2}mR^2$. The cylinder is released from rest at a height h on an inclined plane, and rolls down the plane without slipping. What is the velocity v of the cylinder when it reaches the bottom of the incline?

a. $\sqrt{\frac{4}{3}gh}$

b. $\sqrt{\frac{1}{2}gh}$

c. $\sqrt{\frac{3}{4}gh}$

d. $\sqrt{\frac{3}{4}mgh}$

e. $\sqrt{gh}$

Answer:

The correct answer is *a*. This is a conservation of energy problem in which the initial gravitational potential energy of the cylinder at the top of the plane is converted to both translational and rotational kinetic energy at the bottom.

$$U_g = K_{translational} + K_{rotational}$$

$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

$$I_{cylinder} = \tfrac{1}{2}mR^2,\ and\ \ \omega = \frac{v}{R}$$

$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2$$

$$gh = \tfrac{3}{4}v^2$$

$$v = \sqrt{\tfrac{4}{3}gh}$$

Question:

A large disk, with mass M and radius R, is released from the top of an incline of height h. The disk has a moment of inertia $\frac{1}{2}MR^2$, and rolls without slipping down the incline. Its speed at the bottom is:

a. $\sqrt{\frac{4}{3}gh}$

b. $\sqrt{2gh}$

c. $\sqrt{\frac{10}{7}gh}$

d. $\sqrt{\frac{5}{2}gh}$

e. $\sqrt{gh}$

Answer:

The correct answer is *a*. This is a Conservation of Energy problem, in which the potential energy U at the top of the ramp is converted to both $K_{\text{translational}}$ and $K_{\text{rotational}}$ at the bottom of the ramp.

$$U = K_{translational} + K_{rotational}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$v = r\omega$, so

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2$$

$$v = \sqrt{\frac{4}{3}gh}$$

Question:

"Moment of inertia" is the rotational equivalent of:

a. velocity
b. momentum
c. torque
d. mass
e. potential energy

Answer:

The correct answer is *d*. "Mass" is a measure of an object's inertia, its resistance to a change in motion. In the same way, "moment of inertia" is a resistance to a change in rotation.

Question:
An object's *inertia* is a measure of how difficult it is to change the object's state of motion. What is *rotational inertia*?

a. A quantity that depends on an object's mass.
b. A quantity that depends on the distribution of an object's mass.
c. A measure of how difficult it is to change an object's state of rotation.
d. The same thing as *moment of inertia*.
e. All of the above

Answer:
The correct answer is *e*. The rotational inertia of an object is also called its "moment of inertia," and is a measure of how difficult it is to get an object rotating, or of how difficult it is to get it to stop rotating (if it's already in motion).

Rotational inertia depends both on the mass of the object and the distribution of that mass. The farther away the mass is, relative to the axis of rotation, the greater the object's rotational inertia.

Take a pencil, attach two blobs of clay to it, and then try to wiggle the pencil back and forth. The pencil with the blobs of clay closer to the axis of rotation is much easier to wiggle because it has a smaller *rotational inertia*.

Question:

A long, thin, rod of with moment of inertia I=2 kg•m^2 is free to rotate about an axis passing through the midpoint of the rod. The rod begins rotating from rest at time t=0 seconds, accelerating constantly so that it has a rotational velocity of 4π rad/s after rotating through two complete revolutions. What is the rod's rotational kinetic energy at this point?

a. $16\pi^2\ J$
b. $32\pi\ J$
c. $8\pi^2\ J$
d. $4\pi^2\ J$
e. $32\pi^2\ J$

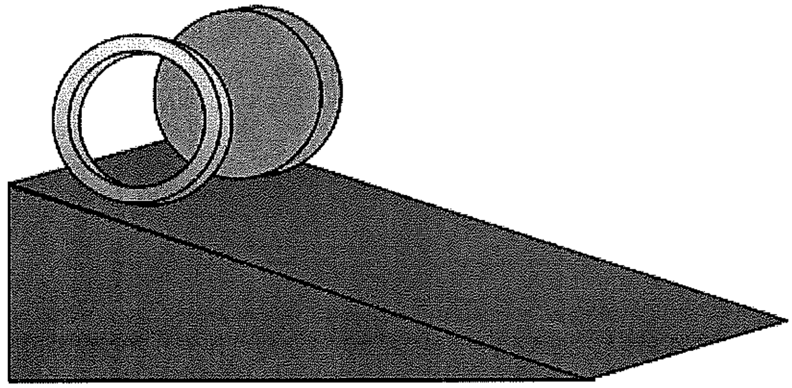
Answer:

The correct answer is *a*. The rod's kinetic energy can be calculated using $K_{rotational} = \frac{1}{2}I\omega^2$.

$$K = \frac{1}{2}I\omega^2$$

$$K = \frac{1}{2}\left(2kg \bullet m^2\right)\left(4\pi\frac{rad}{s}\right)^2$$

$$K = 16\pi^2\ J$$



Question:
A blue hoop and an orange disk—each with the same mass and the same radius—are placed at the top of an inclined plane and released at the same time so that they roll to the bottom. Which object has the greater rotational inertia? Which object reaches the bottom of the incline first?

a. The hoop has the greater rotational inertia, but the disk reaches the bottom first.
b. The hoop has the greater rotational inertia, and the hoop reaches the bottom first.
c. The disk has the greater rotational inertia, but the hoop reaches the bottom first.
d. The disk has the greater rotational inertia, and the disk reaches the bottom first.
e. The hoop and the disk have equal rotational inertias, and reach the bottom at the same time.

Answer:
The correct answer is ***a***. The hoop has all of its mass located at a maximum distance away from the axis of rotation (and thus a greater rotational inertia), while the disk has its mass distributed throughout. The object with the greatest rotational inertia (the hoop) is also the most difficult to get rotating. Because the disk is easier to get rotating, it will accelerate to the bottom of the ramp first.

How one goes about calculating rotational inertia usually requires calculus, which is a bit beyond the scope of this course. It may interest you to know, however, that the "disk versus hoop" result above doesn't depend on mass or radius—*any* disk, of any mass or radius, will beat any hoop in the race to the bottom of the ramp!

Question:

A square of mass M and sides of length L has a moment of inertia I_0 when rotated about an axis perpendicular to its surface and passing through its center, as shown. Now a lump of clay, also of mass M is attached to one corner of the square as shown. What is the new moment of inertia of the masses about the same axis of rotation?

a. $I_0 + \dfrac{ML^2}{4}$

b. $I_0 + \dfrac{ML^2}{2}$

c. $I_0 + \dfrac{\sqrt{2}ML^2}{2}$

d. $I_0 + 2ML^2$

e. $I_0 + ML^2$

Answer:

The correct answer is *b*. The moment of inertia for the system can be calculated by adding the two individual moments of inertia as follows:

$$I_{total} = I_0 + I_{clay}$$

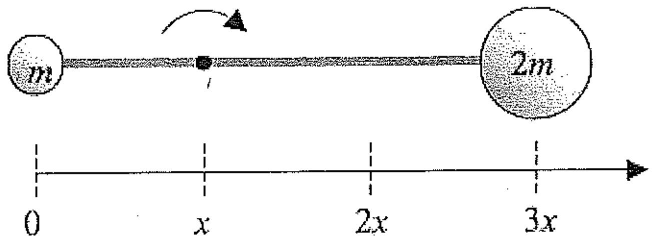
$$I_{clay} = MR^2$$

$$R = \sqrt{2}\frac{L}{2}$$

$$I_{clay} = M\left(\sqrt{2}\frac{L}{2}\right)^2 = \frac{ML^2}{2}$$

$$I_{total} = I_0 + \frac{ML^2}{2}$$

Question:



A solid sphere of mass *m* is fastened to another sphere of mass 2*m* by a thin rod with a length of 3*x*. The spheres have negligible size, and the rod has negligible mass. What is the moment of inertia of the system of spheres as the rod is rotated about the point located at position *x*, as shown?

a. $3\ mx^2$
b. $4\ mx^2$
c. $5\ mx^2$
d. $9\ mx^2$
e. $10\ mx^2$

***Hint: For this case, the solid spheres can be considered point masses with rotational inertia equal to mr^2.**

Answer:

The correct answer is *d*. Moment of inertia for a system of discrete masses is calculated as follows:

$$I = \sum mr^2$$

$$I = m(x)^2 + 2m(2x)^2$$

$$I = mx^2 + 8mx^2 = 9mx^2$$