Question:

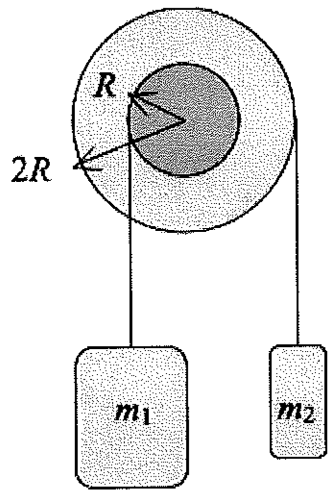
Answer the following questions:

1. What is torque?
2. What angular quantity does the net torque determine?
3. Torque is the rotational analog to what linear quantity?

Answer:

1. Torque is the twisting affect produced by a force about some point.
2. The rate of change of angular momentum.
3. Torque in rotational dynamics is analogous to force in translational dynamics.

Question:



Two solid disks have radii of R and 2R, and are fastened together and placed on a frictionless horizontal axis to form a pulley. The pulley has a total moment of inertia *I*. When two masses, $m_1 > m_2$, are hung from cords attached to the pulley as shown here, what is the angular acceleration of the system?

a. $\frac{Rg(m_1 - 2m_2)}{I}$

b. $\frac{Rg(2m_2 - m_1)}{I}$

c. $\frac{Rg(2m_1 - m_2)}{I}$

d. $\alpha = \frac{Rg(m_1 - 2m_2)}{\text{[redacted]}\ m_1R^2 + 4m_2R^2}$

e. $\alpha = \frac{Rg(m_1 - 2m_2)}{I - m_1R^2 + 4m_2R^2}$

Answer: ? or a

The correct answer is *d*. The angular acceleration of the system can be determined by calculating the sum of the torques ($\tau = r \times F$) and applying this to the rotational form of Newton's Second Law:

$$\tau_{net} = I\alpha, \ so\ \tau_1 - \tau_2 = I\alpha$$

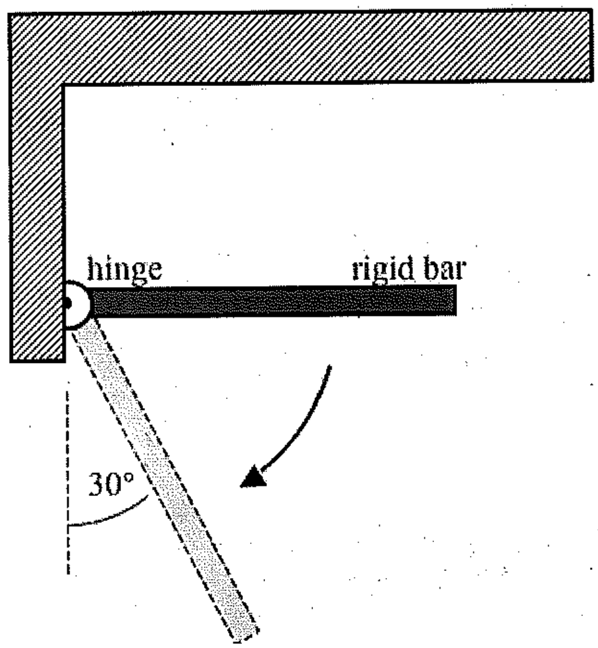
$$F_{net} = ma, \text{ so } m_1g - T_1 = m_1a, \text{ and } T_2 - m_2g = m_2a$$

Rearranging for Tensions, and using $a = r\alpha$

$$T_1 = m_1g - m_1r\alpha \quad and \quad T_2 = m_2r\alpha + m_2g$$

$$R(m_1g - m_1r\alpha) - 2R(m_2r\alpha + m_2g) = I\alpha$$

Question:



A rigid bar with a mass M and length L is free to rotate about a frictionless hinge at a wall. The bar has a moment of inertia $I = 1/3\ ML^2$ about the hinge, and is released from rest when it is in a horizontal position as shown. What is the instantaneous angular acceleration when the bar has swung down so that it makes an angle of 30° to the vertical?

a. $\frac{g}{3L}$ b. $\frac{2g}{3L}$ c. $\frac{g}{L}$ d. $\frac{3g}{4L}$ e. $\frac{3g}{2L}$

Answer:

The correct answer is *d*. The bar is accerlating angularly in response to the torque due to the force of gravity acting on the center of mass. Its angular acceleration due to this torque at the final position can be calculate as follows:

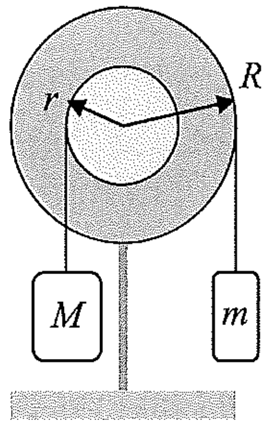
$$\tau = I\alpha$$

$$r \times F = \left(\frac{1}{3}ML^2\right)\alpha$$

$$\frac{L}{2}mg\sin 30 = \left(\frac{1}{3}ML^2\right)\alpha$$

$$\alpha = \frac{3g}{4L}$$

Question:



The pulley system consists of two solid disks of different radii fastened together coaxially, with two different masses connected to the pulleys as shown above. Under what condition will this pulley system be in static equilibrium?

a. $m = M$
b. $rm = RM$
c. $r^2m = R^2M$
d. $rM = Rm$
e. $r^2M = R^2m$

Answer:
The correct answer is *d.* The pulley system will be in equilibrium when the sum of the torques acting on the pulleys is 0.

$$\sum \tau = 0$$

$$\tau_M - \tau_m = 0$$

$$\tau = r \times F$$

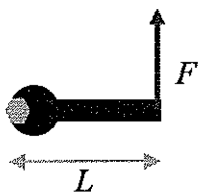
$$(r \times Mg) - (R \times mg) = 0$$

$$rMg = Rmg$$

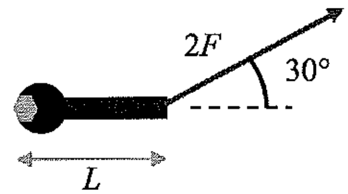
$$rM = Rm$$

Question:
A series of wrenches of different lengths is used on a hexagonal bolt, as shown below. Which combination of wrench length and Force applies the greatest torque to the bolt?

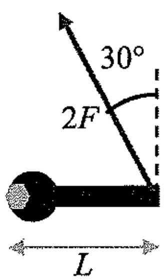
a.



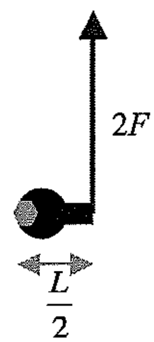
b.



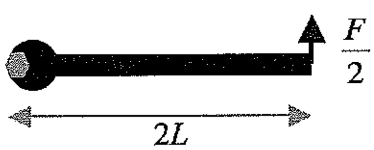
c.



d.



e.

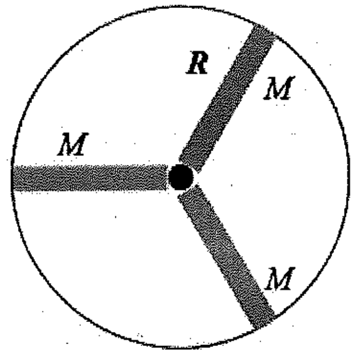


Answer:
The correct answer is *c*. Torque, the "turning effect" produced by a Force applied to a moment-arm, is calculated according to $\tau = \mathbf{r} \times \mathbf{F}$, or $\tau = \mathbf{rF}\sin\theta$, where θ is the angle between the vectors **r** and **F**. Here, each combination of wrench length and Force produces a net torque of LF except for answer *c* :

$$\tau = \mathbf{r} \times \mathbf{F} = rF\sin\theta$$

$$\tau = LF\sin 120 = LF\sin 60 = L2F\tfrac{\sqrt{3}}{2} = LF\sqrt{3}$$

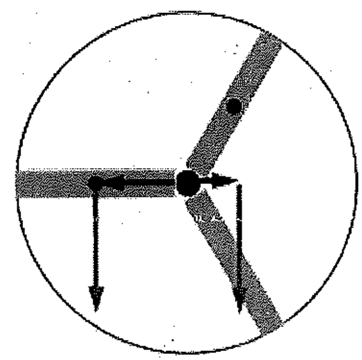
Question:



A wheel consists of a three uniform spokes, with length R and mass M, mounted 120 degrees apart on a horizontal frictionless axle and connected by a rim of negligible mass. Consider the counterclockwise direction to be positive. When the spokes are oriented as shown in the diagram above, the net Torque on the wheel due to the weight of the spokes is

a. $+RM\mathbf{g}$
b. $-2RM\mathbf{g}$
c. $\frac{R}{2}M\mathbf{g}\left(\sqrt{3}-1\right)$
d. $\frac{R}{2}M\mathbf{g}\left(1-\sqrt{3}\right)$
e. 0

Answer:



The correct answer is *d*. Torque is the turning effect produced by forces, and is calculated based on the Force applied at some distance relative to an axis of rotation. The direction of the Force is a factor as well. If we consider the weight of each spoke to be acting at its center of mass (the middle of the spoke), we see that:

For the spoke on the left: $\tau_{counterclockwise} = rF\cos\theta = \frac{R}{2}M\mathbf{g}$

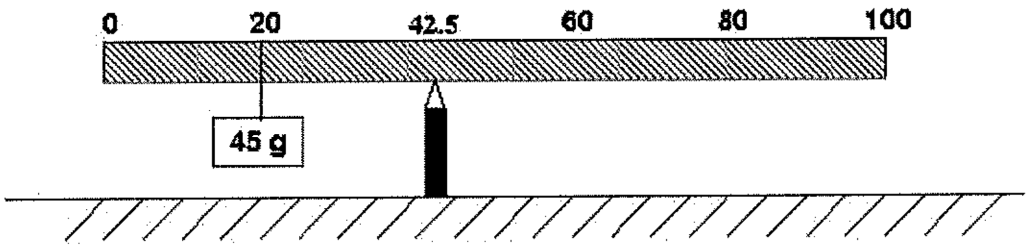
For the spokes (two) on the right: $\tau_{clockwise} = 2\left(rF\cos\theta\right) = 2\left(\frac{R}{2}M\mathbf{g}\frac{\sqrt{3}}{2}\right) = \frac{R}{2}M\mathbf{g}\sqrt{3}$

The net torque, then, is based on

$$\tau_{counterclockwise} + \tau_{clockwise} = \frac{R}{2}M\mathbf{g} + -\left(\frac{R}{2}M\mathbf{g}\sqrt{3}\right) = \frac{R}{2}M\mathbf{g}\left(1-\sqrt{3}\right)$$

Question

A uniform meter stick has a 45.0 g mass placed at the 20 cm mark as shown in the figure. If a pivot is placed at the 42.5 cm mark and the meter stick remains horizontal in static equilibrium, what is the mass of the meter stick?

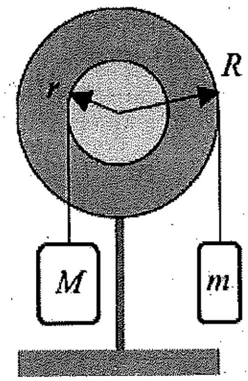


(A) 18.0 g (B) 45.0 g (C) 72.0 g (D) 120.0 g (E) 135.0 g

Answer

E

Question:



The pulley system consists of two solid disks of different radii fastened together coaxially, with two different masses connected to the pulleys as shown above. Under what condition will this pulley system be in static equilibrium?

a. $m = M$
b. $rm = RM$
c. $r^2m = R^2M$
d. $rM = Rm$
e. $r^2M = R^2m$

***Static equilibrium is a condition in which no rotation occurs.**

Answer:
The correct answer is *d*. The pulley system will be in equilibrium when the sum of the torques acting on the pulleys is 0.

$$\sum \tau = 0$$

$$\tau_M - \tau_m = 0$$

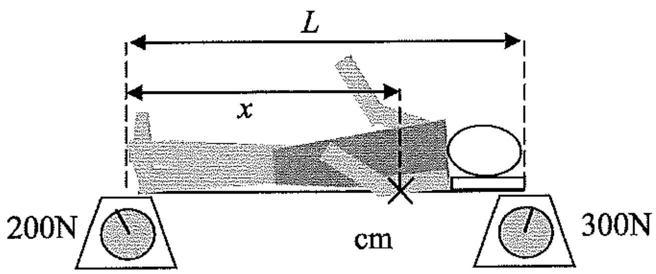
$$\tau = r \times F$$

$$(r \times Mg) - (R \times mg) = 0$$

$$rMg = Rmg$$

$$rM = Rm$$

Question:



A student lies on a rigid platform of negligible mass, which is in turn placed upon two spring scales as shown. The scale on the left, at $x = 0$, reads 200 Newtons, and the scale on the right, at $x = L$, reads 300 Newtons. At what position x is the center of mass located?

a. $\frac{1}{2}L$
b. $\frac{2}{5}L$
c. $\frac{3}{5}L$
d. $\frac{3}{4}L$
e. $\frac{4}{5}L$

Answer:

The correct answer is *c*. Think of this as a torque problem, with the sum of torques about the center of mass adding up to zero.

$$\sum \tau = 0$$

$$\tau_{feet} - \tau_{head} = 0$$

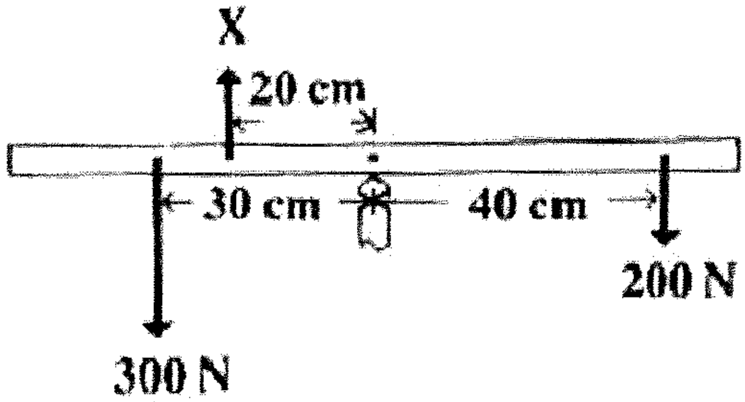
$$r \times F_{feet} = r \times F_{head}$$

$$x(200N) = (L - x)(300N)$$

$$200x + 300x = 300L$$

$$x = \frac{3}{5}L$$

Question



A uniform meterstick is balanced at its midpoint with several forces applied as shown below. If the stick is in equilibrium, the magnitude of the force X in newtons (N) is

(A) 50 N (B) 100 N (C) 200 N (D) 300 N

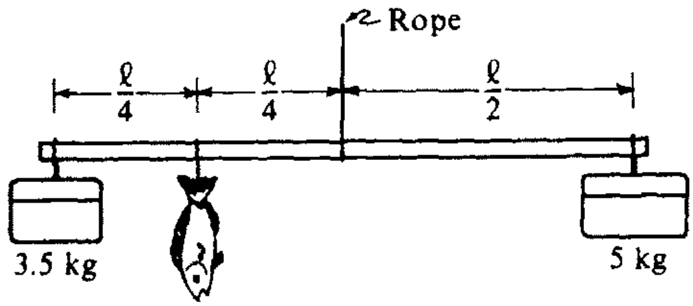
(E) impossible to determine without the weight of the stick

Answer

A

Question

To weigh a fish, a person hangs a tackle box of mass 3.5 kilograms and a cooler of mass 5 kilograms from the ends of a uniform rigid pole that is suspended by a rope attached to its center. The system balances when the fish hangs at a point 1/4 of the rod's length from the tackle box. What is the mass of the fish?

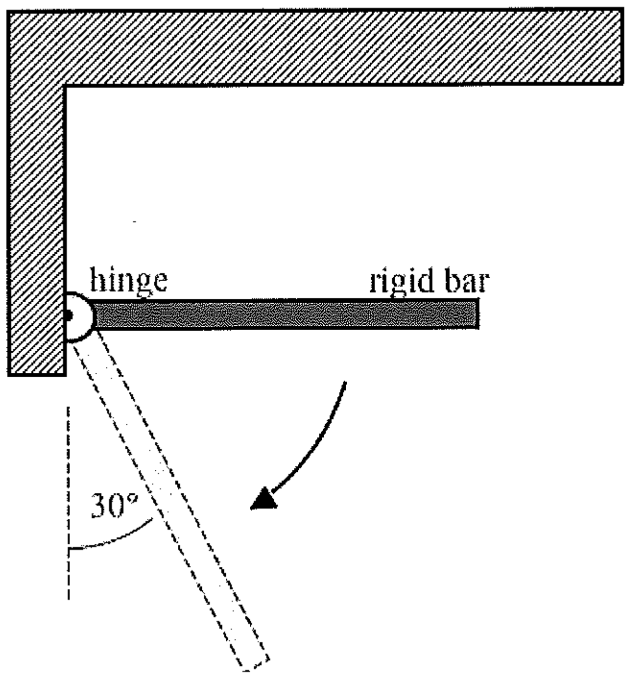


(A) 1.5 kg (B) 2 kg (C) 3 kg (D) 6 kg (E) 6.5 kg

Answer

C

Question:



A rigid bar with a mass M and length L is free to rotate about a frictionless hinge at a wall. The bar has a moment of inertia $I = 1/3\ ML^2$ about the hinge, and is released from rest when it is in a horizontal position as shown. What is the instantaneous angular acceleration when the bar has swung down so that it makes an angle of 30° to the vertical?

a. $\dfrac{g}{3L}$ b. $\dfrac{2g}{3L}$ c. $\dfrac{g}{L}$ d. $\dfrac{3g}{4L}$ e. $\dfrac{3g}{2L}$

Answer:

The correct answer is *d*. The bar is accerlating angularly in response to the torque due to the force of gravity acting on the center of mass. Its angular acceleration due to this torque at the final position can be calculate as follows:

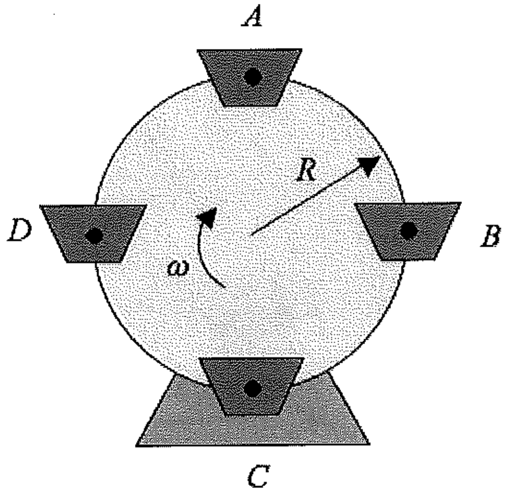
$$\tau = I\alpha$$

$$r \times F = \left(\frac{1}{3}ML^2\right)\alpha$$

$$\frac{L}{2}mg\sin 30 = \left(\frac{1}{3}ML^2\right)\alpha$$

$$\alpha = \frac{3g}{4L}$$

Question:



A large Ferris wheel at an amusement park has four seats, located 90° from each other and at a distance R from the axis. Each seat is attached to the wheel by a strong axle. As the Ferris wheel rotates with a constant angular velocity ω, the seats move past positions A, B, C, and D as shown.

What force must an axle provide to keep a seat of mass m moving past position A?

a. $\frac{1}{2}m\omega^2$
b. $mg - mr\omega^2$
c. $mg + mr\omega^2$
d. $mg - mr\omega$
e. mg

Answer:

The correct answer is *b*. Draw a free-body diagram for the seat at the top of the wheel, noting that the Force due to gravity down must be greater than the Force of the axle up: the vector sum of these two forces provides the centripetal force that keeps the seat moving in a circular path. Then do a centripetal force analysis using Newton's Second Law to get the answer:

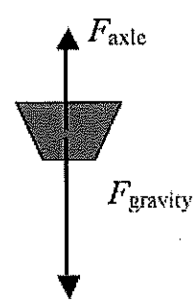
$$F_c = \frac{mv^2}{r}$$

$$F_{gravity} - F_{axle} = \frac{mv^2}{r}$$

$$F_{axle} = mg - \frac{mv^2}{r}$$

$$v = r\omega$$

$$F_{axle} = mg - mr\omega^2$$



Question:

A horizontally-mounted disk with moment of inertia I spins about a frictionless axle. At time t=0, the initial angular speed of the disk is ω. A constant torque τ is applied to the disk, causing it to come to a halt in time t. How much Power is required to dissipate the wheel's energy during this time?

a. $\dfrac{I\omega^2}{2t}$

b. $\dfrac{I\omega}{t}$

c. $\dfrac{I\omega^2}{2\tau}$

d. $\dfrac{I\omega}{\tau}$

e. $\dfrac{I\omega^2}{2}$

Answer:

The correct answer is ***a***. The Power required to dissipate the wheel's initial energy is calculated using $P=W/t$, where W is the Work required to change the wheel's kinetic energy from its initial value to 0:

$$P = \frac{W}{t}$$

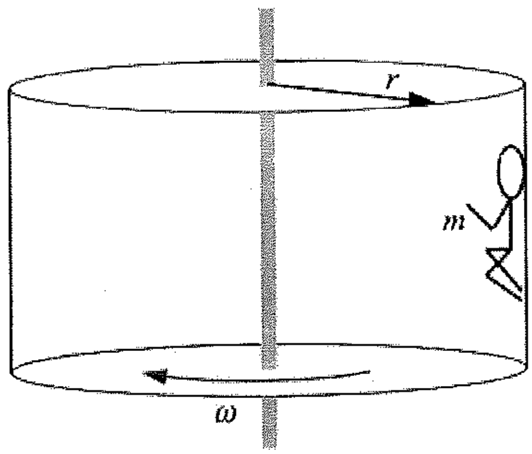
$$W = \Delta K$$

$$\Delta K = \tfrac{1}{2}I\omega^2 - \tfrac{1}{2}I\omega_0{}^2$$

$$W = 0 - \tfrac{1}{2}I\omega_0{}^2$$

$$P = \frac{\tfrac{1}{2}I\omega_0{}^2}{t} = \frac{I\omega^2}{2t}$$

Question:



A ride at an amusement park consists of a hollow cylinder with a student placed against the wall as shown. When the cylinder rotates quickly enough, the student is able to lift her feet off the floor and remain stuck to the wall. In terms of the student's mass m, the radius of the cylinder r, the coefficient of static friction μ between the student and the wall, and fundamental quantities, determine the minimum rotational velocity ω that the ride can have while still allowing the student to stick to the wall and not slide down.

a. $\sqrt{\frac{g}{\mu r}}$ b. μmg c. $\sqrt{r\mu g}$ d. $\sqrt{\frac{\mu g}{r}}$ e. μrg

Answer:

The correct answer is *a*. A free-body analysis of the student against the wall of the cylinder shows that the friction Force *f* must be equal to the force of gravity *mg*, and that the normal force F_{Normal} acts as a centripetal force to keep the student moving in a circle.

Putting those pieces together, then:

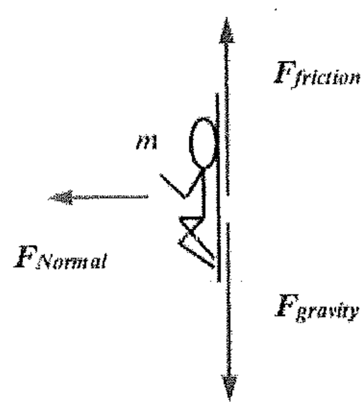
$$\sum F_x = \frac{mv^2}{r} = F_N$$

$$\sum F_y = F_{friction} - F_{gravity} = 0,\ so\ F_f = mg$$

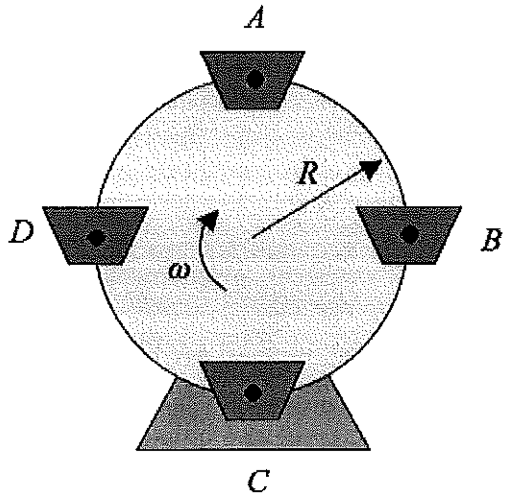
$$F_f = \mu F_N\ and\ v = r\omega$$

Solve to get:

$$\omega = \sqrt{\frac{g}{\mu r}}$$



Question:



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What force must an axle provide to keep a seat of mass m moving past position A?

a. $\frac{1}{2}m\omega^2$
b. $mg - mr\omega^2$
c. $mg + mr\omega^2$
d. $mg - mr\omega$
e. mg

Answer:

The correct answer is *b*. Draw a free-body diagram for the seat at the top of the wheel, noting that the Force due to gravity down must be greater than the Force of the axle up: the vector sum of these two forces provides the centripetal force that keeps the seat moving in a circular path. Then do a centripetal force analysis using Newton's Second Law to get the answer:

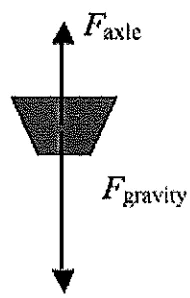
$$F_c = \frac{mv^2}{r}$$

$$F_{gravity} - F_{axle} = \frac{mv^2}{r}$$

$$F_{axle} = mg - \frac{mv^2}{r}$$

$$v = r\omega$$

$$F_{axle} = mg - mr\omega^2$$



Question:

A long, thin, rod of with moment of inertia I=2 kg•m^2 is free to rotate about an axis passing through the midpoint of the rod. The rod begins rotating from rest at time t=0 seconds, accelerating constantly so that it has a rotational velocity of 4π rad/s after rotating through two complete revolutions. What is the rod's angular momentum at this point?

a. $2\pi \;\; kg \bullet m^2/s$
b. $8\pi \;\; kg \bullet m^2/s$
c. $8 \;\; kg \bullet m^2/s$
d. $4\pi \;\; kg \bullet m^2$
e. $4 \;\; kg \bullet m^2$

Answer:

The correct answer is *b*. The angular momentum of the rod can be calculated using $L = I\omega$:

$$L = I\omega$$

$$L = (2kg \bullet m^2)(4\pi \;\; rad/s)$$

$$L = 8\pi \;\; kg \bullet m^2/s$$

Question:

A solid disk with radius R, mass M, and moment of inertia $I = \frac{1}{2}MR^2$, rolls along a surface (without slipping) at constant velocity v. What is the angular momentum of the disk about its own axis as it rolls?

a. MRv

b. $\frac{MRv}{2}$

c. $2MRv$

d. $M\omega$

e. $\frac{M\omega}{2}$

Answer:

The correct answer is ***b***. The angular momentum can be calculated as follows:

$$I = \frac{1}{2}MR^2$$

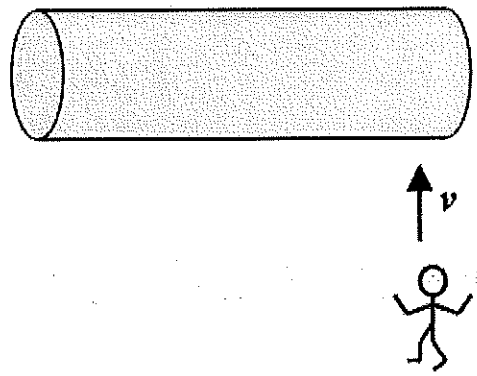
$$\omega = \frac{v}{r}$$

$$L = I\omega$$

$$L = \left(\frac{1}{2}MR^2\right)\left(\frac{v}{r}\right)$$

$$L = \frac{MRv}{2}$$

Question:



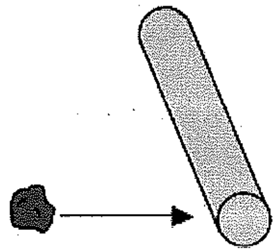
An astronaut is traveling with an initial velocity v towards one end of a cylindrical space station, which is considered to be floating essentially stationary in space. The astronaut arrives at the space station and grabs hold of it, causing the space station to begin moving. Which of the following statements regarding the system of the astronaut–space station is *false*?

a. In the collision, the linear momentum of the system is conserved.
b. Before the collision, the astronaut has an angular momentum relative to the center of the space station.
c. In the collision, the angular momentum of the system is conserved.
d. In the collision, the kinetic energy of the system is conserved.
e. After the collision, the space station has a linear momentum.

Answer:
The correct answer is ***d***. The kinetic energy of the system is ***not*** conserved, as some energy is dissipated as heat when the astronaut collides with and grabs hold of the space station, in a ***perfectly inelastic*** collision.

The other statements are all true. The astronaut has both a linear momentum $\mathbf{p} = m\mathbf{v}$ and an angular momentum $\mathbf{L} = \mathbf{r} \times m\mathbf{v}$ before and after the collision, as does the space station. These quantities in the system are conserved during the collision, because any forces or torques between the two bodies are internal to our defined system. With no significant external Force, $F_{ext} = \frac{d\mathbf{p}}{dt} = 0$, and with no external torque, $\tau_{ext} = \frac{d\mathbf{L}}{dt} = 0$. After the collision, the astronaut–space shuttle system will move through space with both a rotational velocity and a linear velocity, according to the conservations of linear and angular momentum.

Question:



An asteroid traveling through space collides with one end of a long, cylindrical satellite as shown above, and sticks to the satellite. Which of the following is true of the isolated asteroid-satellite system in this collision?

a. Kinetic energy K is conserved
b. Total Energy E is conserved, but angular momentum **L** is not conserved
c. Angular momentum **L** is conserved, but linear momentum **p** is not conserved
d. Angular momentum **L** is conserved, and total energy E is conserved
e. Linear momentum **p** is conserved but angular momentum **L** is not conserved

Answer:
The correct answer is *d.* For isolated systems, all three conservation laws are always in effect: total energy is conserved (although kinetic energy K is not conserved in this perfectly inelastic collsion), linear momentum is conserved, and angular momentum is conserved.

Question:

A certain star, of mass m and radius r, is rotating with a rotational velocity ω. After the star collapses, it has the same mass but with a much smaller radius. Which statement below is true?

a. The star's moment of inertia I has decreased, and its angular momentum L has increased.
b. The star's moment of inertia I has decreased, and its angular velocity ω has decreased.
c. The star's moment of inertia I remains constant, and its angular momentum L has increased.
d. The star's angular momentum L remains constant, and its rotational kinetic energy has decreased.
e. The star's angular momentum L remains constant, and its rotational kinetic energy has increased.

Answer:

The correct answer is *e*. According to conservation of angular momentum, the angular momentum L of the star remains constant, so when its moment of inertia I increases (due to the decreased radius), its angular velocity ω goes up proportionally, according to:

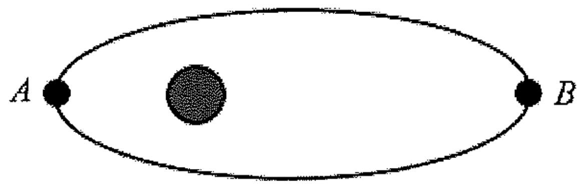
$$L_{initial} = L_{final}$$

$$I_i\omega_i = I_f\omega_f$$

$$\omega_f = \frac{I_i}{I_f}\omega_i$$

The star's rotational kinetic energy, based on $K_{rotational} = \frac{1}{2}I\omega^2$ also goes up. Although I has decreased, $K_{rotational}$ increases with the *square* of ω, leading to a net increase in energy.

Question:

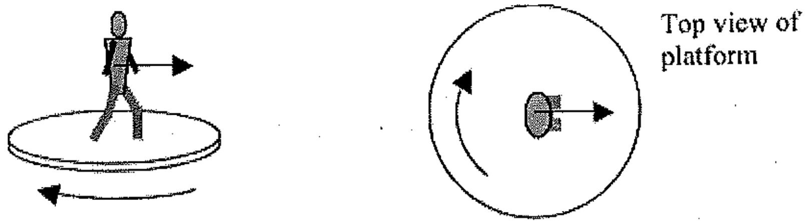


A comet orbits the earth in an elliptical path as shown. Which of the following is true for the magnitudes of the linear momentum **p** and angular momentum **L** for the comet at positions *A* and *B*?

a. $p_A < p_B$; $L_A > L_B$
b. $p_A = p_B$; $L_A = L_B$
c. $p_A > p_B$; $L_A = L_B$
d. $p_A > p_B$; $L_A > L_B$
e. $p_A = p_B$; $L_A > L_B$

Answer:
The correct answer is *c*. With no external force acting on the system, the angular momentum $\mathbf{L} = \mathbf{r} \times m\mathbf{v}$ of the comet will remain constant. The speed of the comet is greater at position *A*, however, so the magnitude of the linear momentum at that position will be greater than it is at position *B*.

Question:

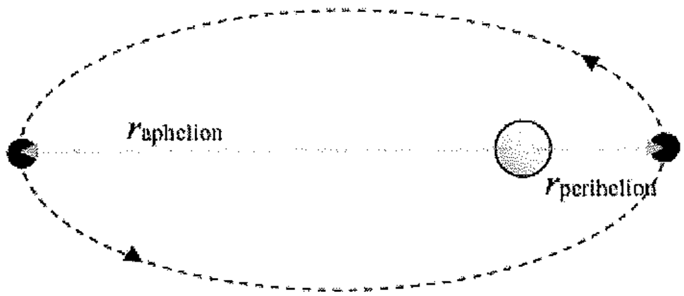


A freely-rotating merry-go-round has a child standing at the center of the platform. As the child walks toward the outer edge of the platform, which statement is true?

a. Angular momentum increases and rotational kinetic energy increases.
b. Angular momentum decreases and rotational kinetic energy decreases.
c. Angular momentum stays constant and rotational kinetic energy stays constant.
d. Angular momentum stays constant and rotational kinetic energy increases.
e. Angular momentum stays constant and rotational kinetic energy decreases.

Answer:
The correct answer is *e*. With no external source of torque, the angular momentum of the system remains constant. A conservation of energy analysis reveals that, although the moment of inertia I is increasing as the child walks to the edge, I and ω are inversely related, and the kinetic energy also changes as the *square* of ω. Thus, $K_{rotational} = \frac{1}{2}I\omega^2$, with ω decreasing, is also decreasing in magnitude.

Question:



A planet has an elliptical orbit around a large star, as shown. If the planet has a speed of $v_{aphelion}$ at its farthest distance from the star, which of the following statements about the planets velocity at perihelion is true?

a. $v_{perihelion} < v_{aphelion}$, because the angular momentum of the system has decreased
b. $v_{perihelion} < v_{aphelion}$, because the angular momentum of the system is the same
c. $v_{perihelion} > v_{aphelion}$, because the angular momentum of the system has increased
d. $v_{perihelion} > v_{aphelion}$, because the angular momentum of the system is the same
e. $v_{perihelion} = v_{aphelion}$, because the angular momentum of the system is the same

Answer:
The correct answer is *d*. Angular momentum is conserved in this problem, so we can say:

$$L_i = L_f$$

$$r_{aphelion} \times mv_{aphelion} = r_{perihelion} \times mv_{perihelion}$$

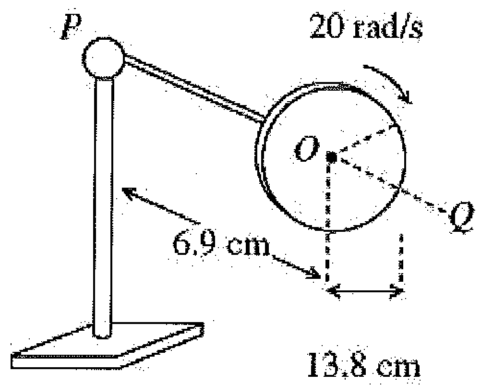
$r \times v = rv$ when r and v are perpendicular

$$v_{perihelion} = \frac{r_{aphelion}}{r_{perihelion}} v_{aphelion}$$

Because $r_{aphelion} > r_{perihelion}$, we know that the velocity is greater at perihelion.

Question:

In the figure below, the rotor of a gyroscope is a uniform disk. The ball pivot at ***P*** is frictionless. At a given instant, the rotor shaft is horizontal and the rotor is rotating with an angular velocity about its axis ***OP***, as shown. The rotor is viewed from point ***Q*** on the axis. The direction of precession of point ***O***, as seen from point ***Q***, is



A) Leftward
B) Upward
C) Rightward
D) Downward
E) None of the above. Since there is no external torque the gyroscope will not undergo precession.

Answer:

A > The rotor's angular momentum is directed into and perpendicular to the page. The direction of the external torque produced by the rotor's weight, by the RH rule, is to the right..causing the rotor's angular momentum to change in this direction. As viewed from the top, the rotor would precess to the left. As seen from point *Q*, point *O* will move to the left.