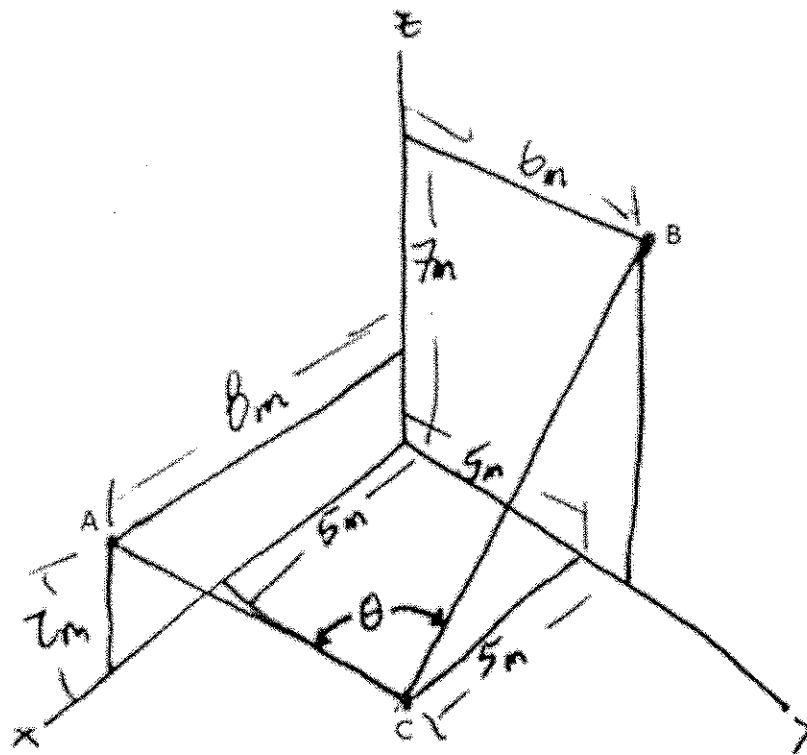


AP Physics C

Instructor: Mr. Butler

Finding the Angle Between Two Vectors

By Scalar (DOT) Product



$$\hat{A} \cdot \hat{B} = AB \cos(\theta) = A_x B_x + A_y B_y + A_z B_z$$

$$\theta = \cos^{-1} \left( \frac{\hat{A} \cdot \hat{B}}{AB} \right) = \cos^{-1} \left( \frac{A_x B_x + A_y B_y + A_z B_z}{AB} \right)$$

$$\theta = \cos^{-1} \left( \frac{(8-5)(0-5) + (0-5)(6-5) + (2-0)(7-0)}{\sqrt{(8-5)^2 + (0-5)^2 + (2-0)^2} * \sqrt{(0-5)^2 + (6-5)^2 + (7-0)^2}} \right)$$

$$\theta = \cos^{-1} \left( \frac{-6}{6.16 * 8.66} \right)$$

$$\theta = 96.5^\circ$$

## By Vector (CROSS) Product

### How to Find the Sine of the Angle Between Two Vectors?

**Example:** Use the cross product to find the sine of the angle between the vectors  $\mathbf{u} = (2, 3, -6)$  and  $\mathbf{v} = (2, 3, 6)$ .

**Solution:**

We can use equation (6) derived from the Lagrange's identity.

$$\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\| \|\mathbf{v}\| \sin(\theta)$$
$$\sin(\theta) = \frac{\|\mathbf{u} \times \mathbf{v}\|}{\|\mathbf{u}\| \|\mathbf{v}\|}$$

**Note 1:** Lagrange's identity:  $\|\mathbf{u} \times \mathbf{v}\|^2 = \|\mathbf{u}\|^2 \|\mathbf{v}\|^2 - (\mathbf{u} \cdot \mathbf{v})^2$ .

**Note 2:** If  $\theta$  denotes the angle between two nonzero vectors  $\mathbf{u}$  and  $\mathbf{v}$ , then  $\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \|\mathbf{v}\| \cos(\theta)$ . (From previous module.)

### How to Find the Sine of the Angle Between Two Vectors? (Cont.)

**Step 1:** Find the cross product of  $\mathbf{u}$  and  $\mathbf{v}$ .

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & -6 \\ 2 & 3 & 6 \end{vmatrix} = \begin{vmatrix} 3 & -6 \\ 3 & 6 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 2 & -6 \\ 2 & 6 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 2 & 3 \\ 2 & 3 \end{vmatrix} \mathbf{k}$$
$$= 36\mathbf{i} - 24\mathbf{j} + 0\mathbf{k} = (36, -24, 0)$$

**Step 2:** Find the norm of the cross product of  $\mathbf{u}$  and  $\mathbf{v}$ .

$$\|\mathbf{u} \times \mathbf{v}\| = \sqrt{(36)^2 + (-24)^2 + (0)^2} = \sqrt{1872}$$

### How to Find the Sine of the Angle Between Two Vectors? (Cont.)

**Step 3:** Find  $\sin(\theta)$ .

$$\sin(\theta) = \frac{\|\mathbf{u} \times \mathbf{v}\|}{\|\mathbf{u}\| \|\mathbf{v}\|}$$
$$= \frac{\sqrt{1872}}{\sqrt{(2)^2 + (3)^2 + (-6)^2} \sqrt{(2)^2 + (3)^2 + (6)^2}}$$
$$= \frac{\sqrt{1872}}{\sqrt{49} \sqrt{49}} = \frac{\sqrt{1872}}{49}$$