Collisions: Elastic and Inelastic Problem Derivations

- Collisions may be separated into several categories, some of which are easier to solve than others:
  - **Completely inelastic** collisions involve objects which stick together afterwards. Kinetic energy is not conserved, but the result is easy to calculate via conservation of momentum.
  - **Partially inelastic** collisions involve objects which separate after they collide, but which are deformed in some way by the interaction. Kinetic energy is not conserved. It's not easy to figure out what happens afterwards, because there are many possible solutions which satisfy conservation of momentum.
  - **Elastic** collisions involve objects which separate after they collide, and which are not changed at all by the interaction. Billiard balls, ping-pong balls, and other hard objects may collide elastically. Kinetic energy *is* conserved in elastic collisions. One must use both conservation of energy *and* conservation of momentum to figure out the motions of the objects afterwards. This usually involves solving 2 equations for 2 unknowns.

Elastic vs. Inelastic Collisions

Physicists divide collisions into several categories :

Completely Inelastic :	bodies stick together KE <u>not</u> conserved
Partially Inelastic :	bodies separate KE <u>not</u> conserved
Elastic :	bodies separate KE is conserved

Completely inelastic collisions are easy to solve : just use conservation of momentum.

Elastic collisions – which occur when hard, rigid objects like marbles or billiard balls collide – take more work. One must use both

conservation of momentum
<u>and</u>
conservation of energy

Why Completely Inelastic is Easy

In many collisions, the mass of each object doesn't change. One is often given the masses and initial velocities ...

	before	after	
Completely Inelastic	m_1 v_{1i} m_2 v_{2i}	m_1 m_2	v_f 1 unknown
Partially Inelastic	m_1 v_{1i} m_2 v_{2i}	m_1 m_2	v_{1f} v_{2f} 2 unknowns
Elastic	m_1 v_{1i} m_2 v_{2i}	m_1 m_2	v_{1f} v_{2f} 2 unknowns

Challenge Question

Crazy Ellie (50 kg) decides to learn bullfighting. She stands her ground as El Toro (900 kg) approaches at $v_i = 8$ m/s. She doesn't manage to sidestep, and is carried back by the bull.

How fast do Ellie and Toro move together?

A. 7.0 m/s
B. 7.6 m/s
C. 7.8 m/s
D. 8.0 m/s

Use momentum to find the answer:

	before	after
Ellie	0	$(50\,\text{kg})\,v_f$
Toro	$(900\,\text{kg})(8\,\text{m/s})$	$(900\,\text{kg})\,v_f$
total	$7200\,\frac{\text{kg}\cdot\text{m}}{\text{s}}$	$(950\,\text{kg})\,v_f$

Conservation of Momentum requires that

$$v_f = \frac{7200\,\frac{\text{kg}\cdot\text{m}}{\text{s}}}{950\,\text{kg}}$$

$$= 7.6\,\text{m/s}$$

This situation – in which the objects stick together after colliding – is called completely inelastic. Note that kinetic energy is <u>not</u> conserved:

Ellie	0 J	1,444 J
Toro	28,800 J	25,992 J
total	28,800 J	27,436 J

Ex of Elastic Collision :

Two billiard balls ($m = 0.2\,kg$) are both moving to the right. But the 8-ball is going faster than the 2-ball:

$$v_8 = 3\ m/s$$
$$v_2 = 1\ m/s$$

What happens when they hit each other?

	before		after	
	KE	p	KE	p
8-ball	$0.9\,J$	$0.6\ \frac{kg \cdot m}{s}$	$(0.1\,kg)\,v_8^2$	$(0.2\,kg)\,v_8$
2-ball	$0.1\,J$	$0.2\ \frac{kg \cdot m}{s}$	$(0.1\,kg)\,v_2^2$	$(0.2\,kg)\,v_2$
total KE	$1.0\,J$		$(0.1\,kg)(v_8^2 + v_2^2)$	
total p		$0.8\ \frac{kg \cdot m}{s}$		$(0.2\,kg)(v_8 + v_2)$

Conservation of Energy demands

$$(0.1\,\text{kg})v_8^2 + (0.1\,\text{kg})v_2^2 = 1.0\,\text{J}$$

Conservation of Momentum demands

$$(0.2\,\text{kg})v_8 + (0.2\,\text{kg})v_2 = 0.8\,\frac{\text{kg}\cdot\text{m}}{\text{s}}$$

Uh-oh. We have two unknowns, v_8 and v_2, the speeds of the two balls after the collision.

Fortunately, we have two equations, so we can solve for both.

From Momentum Eqn:

$$v_2 = \frac{0.8\,\frac{\text{kg}\cdot\text{m}}{\text{s}} - (0.2\,\text{kg})v_8}{0.2\,\text{kg}}$$

$$= 4\,\frac{\text{m}}{\text{s}} - v_8$$

Now substitute this into the Kinetic Energy Eqn:

$$(0.1\,\text{kg})v_8^2 + (0.1\,\text{kg})\left[4\,\frac{\text{m}}{\text{s}} - v_8\right]^2 = 1.0\,\text{J}$$

This is 1 equation in 1 unknown —
we can figure out v_8:

$$v_8^2 + \left[16\,\tfrac{m^2}{s^2} - 8\,\tfrac{m}{s}\,v_8 + v_8^2\right] = 10\,\tfrac{m^2}{s^2}$$

$$2v_8^2 - \left(8\,\tfrac{m}{s}\right)v_8 + 16\,\tfrac{m^2}{s^2} = 10\,\tfrac{m^2}{s^2}$$

$$v_8^2 - \left(4\,\tfrac{m}{s}\right)v_8 + 3\,\tfrac{m^2}{s^2} = 0$$

This is a quadratic equation;
solve it:

$$v_8 = \frac{4\,\frac{m}{s} \pm \sqrt{16\,\frac{m^2}{s^2} - 12\,\frac{m^2}{s^2}}}{2}$$

$$= \frac{4\,\frac{m}{s} \pm 2\,\frac{m}{s}}{2}$$

$$= \begin{cases} 3\,\frac{m}{s} & \text{before collision} \\ 1\,\frac{m}{s} & \text{\underline{after} collision} \end{cases}$$

So $v_8 = 1\ \mathrm{m/s}$

$$v_2 = 4\ \mathrm{m/s} - v_8 = 3\ \mathrm{m/s}$$

Summary of 1-D collisions

Completely Inelastic :

- objects stick together
- use Cons of Momentum
- solution unique (& easy)

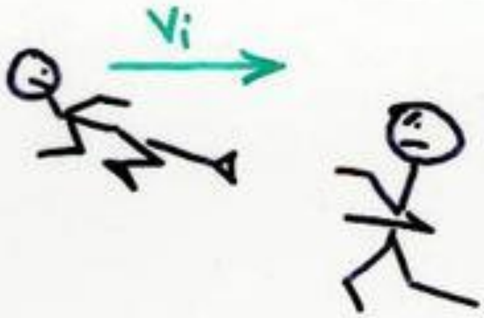
Completely Elastic :

- objects bounce apart
- use Cons of Momentum
  and Cons of Energy
- solution unique (& hard)

Partially Inelastic :

- objects bounce apart but deform / transform
- use Cons of Momentum
- solution depends on how much KE is lost

Challenge Problem (take 2)



Bruce Lee ($m_B = 60\ kg$) flies through the air at $v_i = 6\ m/s$. He kicks Generic Villain ($m_G = 100\ kg$), who stands still. After the two men separate, 20% of the initial Kinetic Energy has disappeared (goes into breaking 3 ribs).

What is Bruce's final velocity?

A. $-0.8\ m/s$

B. $4.3\ m/s$

C. $5.3\ m/s$

D. $6.3\ m/s$

Two identical balls. Ball A has initial speed 10 m/s, Ball B is at rest. They collide. What happens?

	final KE / orig KE	final V_A	final V_B
Completely Elastic	100%	0 m/s	10 m/s
Partially Inelastic	80%	1.13	8.87
Partially Inelastic	60%	2.76	7.24
Completely Inelastic	50%	5	5

Note that even the most inelastic collision still leaves <u>half</u> the initial KE (and <u>all</u> the original momentum).