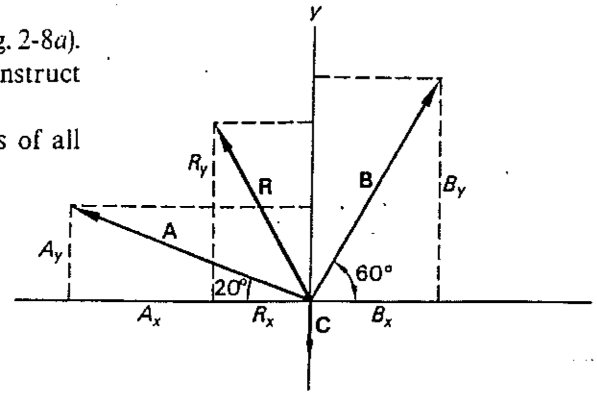
Composition by Rectangular Resolution

(Finding the resultant by resolving vectors into components)

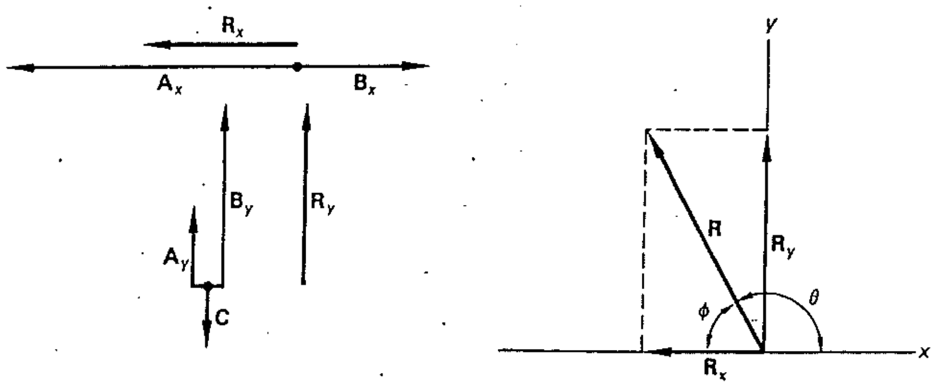
The following procedure of adding vectors to find a resultant may be used. The case shown below considers this procedure for the addition of vectors ***A, B,*** and ***C*** in Fig. 2-8*a*.

1. Draw all vectors from the origin on a set of rectangular coordinates (Fig. 2-8*a*).
2. Resolve all vectors into their x and y components. (You may wish to construct a table of components, as in Example 2-8, below.)
3. Find the x component of the resultant by adding the x components of all the vectors.

$$R_x = A_x + B_x + C_x$$



(a)



(b) (c)

Fig. 2-8 The method of rectangular resolution.

Find the y component of the resultant by adding the y components of all the vectors.

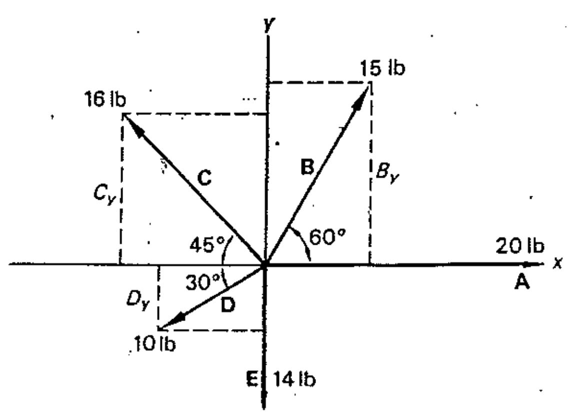
$$R_y = A_y + B_y + C_y$$

4. Obtain the magnitude and direction of the resultant from the two perpendicular vectors $\mathbf{R}_x$ and $\mathbf{R}_y$.

$$\tan\theta = \frac{R_y}{R_x} \qquad R = \sqrt{R_x^2 + R_y^2}$$

Steps 3 and 4 are shown graphically in Fig. 2-8*b* and *c*.

Example 2-8 A trainer holds five horses with reins. The forces they exert on the trainer can be represented by five vectors: $\mathbf{A} = (20\text{ lb}, 0)$, $\mathbf{B} = (15\text{ lb}, 60°)$, $\mathbf{C} = (16\text{ lb}, 135°)$, $\mathbf{D} = (10\text{ lb}, 210°)$, and $\mathbf{E} = (14\text{ lb}, 270°)$. In what direction and with what force must a single horse pull if it is to have the same effect on the trainer?



(a)

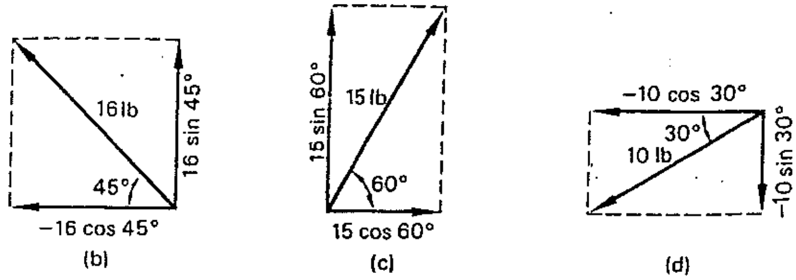


Fig. 2-9 Resolution of all vectors into their x and y components.

Solution Follow the steps described above.

1. Draw a figure representing all forces (Fig. 2-9). Two things should be noticed from the figure: (1) All angles are determined from the x axis, and (2) the components of each vector are labeled and placed opposite and adjacent to known angles.
2. Resolve each force into its x and y components and tabulate as shown in Table 2-3. (Note that force A has no y component and force E has no x component.) Care must be taken to obtain the correct sign for each component from the figure. For instance, C_x, D_x, D_y, and E_y are all negative, and their values must be preceded by a minus sign.
3. Add the x and y components separately to yield R_x and R_y.

Table 2-3

Force	θ_x	x component	y component
$A = 20$ lb	0	$A_x = 20$ lb	$A_y = 0$
$B = 15$ lb	60	$B_x = (15\text{ lb})(\cos 60°)$ $= 7.5$ lb	$B_y = (15\text{ lb})(\sin 60°)$ $= 13.0$ lb
$C = 16$ lb	45	$C_x = (-16\text{ lb})(\cos 45°)$ $= 11.3$ lb	$C_y = (16\text{ lb})(\sin 45°)$ $= 11.3$ lb
$D = 10$ lb	30	$D_x = (-10\text{ lb})(\cos 30°)$ $= -8.66$ lb	$D_y = (-10\text{ lb})(\sin 30°)$ $= -5$ lb
$E = 14$ lb	90	$E_x = 0$	$E_y = -14$ lb
		$R_x = \sum F_x = 7.54$ lb	$R_y = \sum F_y = 5.3$ lb

Since R_x and R_y are known, we have from Fig. 2-10

$$\tan\theta = \frac{R_y}{R_x} = \frac{5.3\text{ lb}}{7.54\text{ lb}} = 0.703$$

or

$$\theta = 35.1°$$

Using this value of θ gives

$$R = \frac{R_y}{\sin\theta} = \frac{5.3\text{ lb}}{\sin 35.1°}$$

or

$$R = \frac{5.3\text{ lb}}{0.575} = 9.22\text{ lb}$$

Therefore a single horse must pull with a force of 9.22 lb at an angle of 35.1°. The magnitude of **R** can also be found from Pythagoras' theorem.

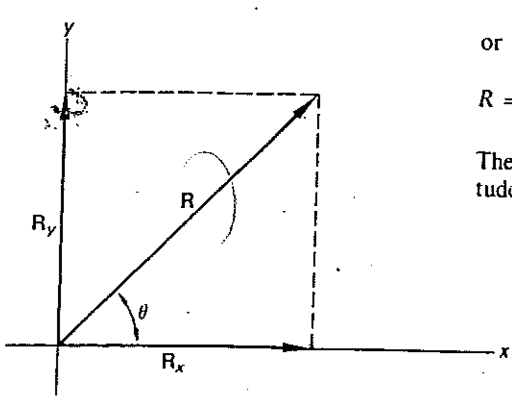


Fig. 2-10 Finding the resultant **R** from its components.