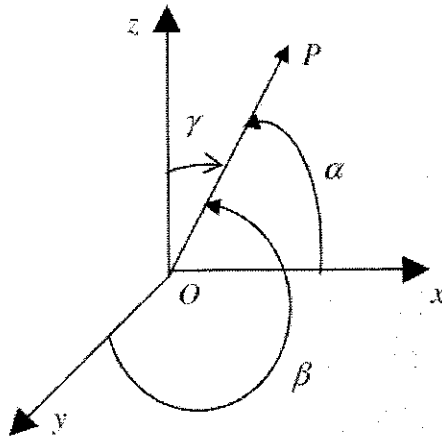


## DIRECTIONS OF ANGLES & DIRECTIONS OF COSINES



$\alpha, \beta, \gamma$  - Are the angles that the vector  $OP$  makes with positive axis  
- Known as the direction angles of vector  $OP$

### DIRECTION OF COSINES

$$\cos \alpha = \frac{x}{|OP|}$$

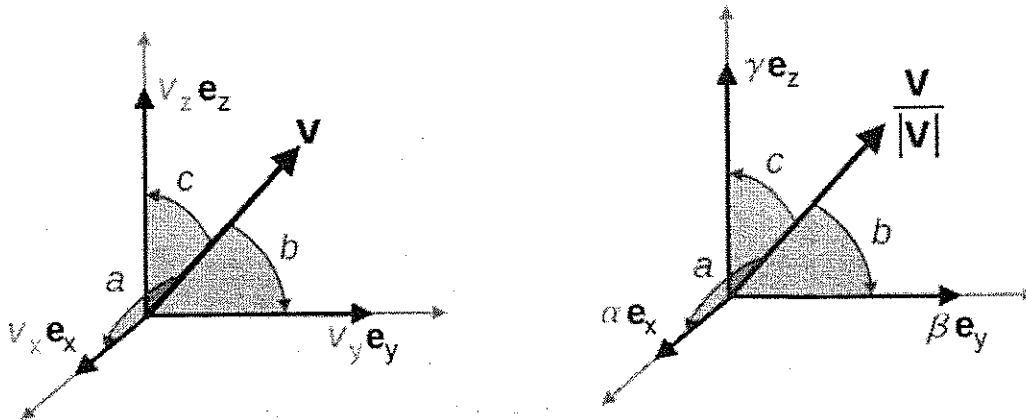
$$\cos \beta = \frac{y}{|OP|}$$

$$\cos \gamma = \frac{z}{|OP|} \quad ; \quad 0^\circ \leq \alpha, \beta, \gamma \leq 180^\circ$$

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The method above employs the application of the magnitude of each axis component of the vector.

An alternate method of finding the axes angles is to apply the DOT product.



If  $\mathbf{v}$  is a Euclidean vector in three-dimensional Euclidean space,  $\mathbb{R}^3$ ,

where  $\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z$  are the standard basis in Cartesian notation, then the direction cosines are

It follows that by squaring each equation and adding the results

Here  $\alpha, \beta$  and  $\gamma$  are the direction cosines and the Cartesian coordinates of the unit vector  $\mathbf{v}/|\mathbf{v}|$ , and  $a, b$  and  $c$  are the direction angles of the vector  $\mathbf{v}$ .

The direction angles  $a, b$  and  $c$  are acute or obtuse angles, i.e.,  $0 \leq a \leq \pi, 0 \leq b \leq \pi$  and  $0 \leq c \leq \pi$ , and they denote the angles formed between  $\mathbf{v}$  and the unit basis vectors,  $\mathbf{e}_x, \mathbf{e}_y$  and  $\mathbf{e}_z$ .

$$\alpha = \cos a = \frac{\mathbf{v} \cdot \mathbf{e}_x}{\|\mathbf{v}\|} = \frac{v_x}{\sqrt{v_x^2 + v_y^2 + v_z^2}},$$

$$\beta = \cos b = \frac{\mathbf{v} \cdot \mathbf{e}_y}{\|\mathbf{v}\|} = \frac{v_y}{\sqrt{v_x^2 + v_y^2 + v_z^2}},$$

$$\gamma = \cos c = \frac{\mathbf{v} \cdot \mathbf{e}_z}{\|\mathbf{v}\|} = \frac{v_z}{\sqrt{v_x^2 + v_y^2 + v_z^2}}.$$