

1

UNITS, PHYSICAL QUANTITIES AND VECTORS

- 1.1. IDENTIFY:** Convert units from mi to km and from km to ft.

SET UP: 1 in. = 2.54 cm, 1 km = 1000 m, 12 in. = 1 ft, 1 mi = 5280 ft.

EXECUTE: (a) $1.00 \text{ mi} = (1.00 \text{ mi}) \left(\frac{5280 \text{ ft}}{1 \text{ mi}} \right) \left(\frac{12 \text{ in.}}{1 \text{ ft}} \right) \left(\frac{2.54 \text{ cm}}{1 \text{ in.}} \right) \left(\frac{1 \text{ m}}{10^2 \text{ cm}} \right) \left(\frac{1 \text{ km}}{10^3 \text{ m}} \right) = 1.61 \text{ km}$

(b) $1.00 \text{ km} = (1.00 \text{ km}) \left(\frac{10^3 \text{ m}}{1 \text{ km}} \right) \left(\frac{10^2 \text{ cm}}{1 \text{ m}} \right) \left(\frac{1 \text{ in.}}{2.54 \text{ cm}} \right) \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right) = 3.28 \times 10^3 \text{ ft}$

EVALUATE: A mile is a greater distance than a kilometer. There are 5280 ft in a mile but only 3280 ft in a km.

- 1.2. IDENTIFY:** Convert volume units from L to in.³.

SET UP: 1 L = 1000 cm³. 1 in. = 2.54 cm

EXECUTE: $0.473 \text{ L} \times \left(\frac{1000 \text{ cm}^3}{1 \text{ L}} \right) \times \left(\frac{1 \text{ in.}}{2.54 \text{ cm}} \right)^3 = 28.9 \text{ in.}^3$

EVALUATE: 1 in.³ is greater than 1 cm³, so the volume in in.³ is a smaller number than the volume in cm³, which is 473 cm³.

- 1.3. IDENTIFY:** We know the speed of light in m/s. $t = d/v$. Convert 1.00 ft to m and t from s to ns.

SET UP: The speed of light is $v = 3.00 \times 10^8 \text{ m/s}$. 1 ft = 0.3048 m. 1 s = 10⁹ ns.

EXECUTE: $t = \frac{0.3048 \text{ m}}{3.00 \times 10^8 \text{ m/s}} = 1.02 \times 10^{-9} \text{ s} = 1.02 \text{ ns}$

EVALUATE: In 1.00 s light travels $3.00 \times 10^8 \text{ m} = 3.00 \times 10^5 \text{ km} = 1.86 \times 10^5 \text{ mi}$.

- 1.4. IDENTIFY:** Convert the units from g to kg and from cm³ to m³.

SET UP: 1 kg = 1000 g. 1 m = 1000 cm.

EXECUTE: $19.3 \frac{\text{g}}{\text{cm}^3} \times \left(\frac{1 \text{ kg}}{1000 \text{ g}} \right) \times \left(\frac{100 \text{ cm}}{1 \text{ m}} \right)^3 = 1.93 \times 10^4 \frac{\text{kg}}{\text{m}^3}$

EVALUATE: The ratio that converts cm to m is cubed, because we need to convert cm³ to m³.

- 1.5. IDENTIFY:** Convert volume units from in.³ to L.

SET UP: 1 L = 1000 cm³. 1 in. = 2.54 cm.

EXECUTE: $(327 \text{ in.}^3) \times (2.54 \text{ cm/in.})^3 \times (1 \text{ L}/1000 \text{ cm}^3) = 5.36 \text{ L}$

EVALUATE: The volume is 5360 cm³. 1 cm³ is less than 1 in.³, so the volume in cm³ is a larger number than the volume in in.³.

- 1.6. IDENTIFY:** Convert ft² to m² and then to hectares.

SET UP: 1.00 hectare = 1.00 × 10⁴ m². 1 ft = 0.3048 m.

EXECUTE: The area is $(12.0 \text{ acres}) \left(\frac{43,600 \text{ ft}^2}{1 \text{ acre}} \right) \left(\frac{0.3048 \text{ m}}{1.00 \text{ ft}} \right)^2 \left(\frac{1.00 \text{ hectare}}{1.00 \times 10^4 \text{ m}^2} \right) = 4.86 \text{ hectares}$.

EVALUATE: Since $1 \text{ ft} = 0.3048 \text{ m}$, $1 \text{ ft}^2 = (0.3048)^2 \text{ m}^2$.

1.7. IDENTIFY: Convert seconds to years.

SET UP: 1 billion seconds = $1 \times 10^9 \text{ s}$. 1 day = 24 h. 1 h = 3600 s.

EXECUTE: 1.00 billion seconds = $(1.00 \times 10^9 \text{ s}) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) \left(\frac{1 \text{ day}}{24 \text{ h}} \right) \left(\frac{1 \text{ y}}{365 \text{ days}} \right) = 31.7 \text{ y}$.

EVALUATE: The conversion $1 \text{ y} = 3.156 \times 10^7 \text{ s}$ assumes $1 \text{ y} = 365.24 \text{ d}$, which is the average for one extra day every four years, in leap years. The problem says instead to assume a 365-day year.

1.8. IDENTIFY: Apply the given conversion factors.

SET UP: 1 furlong = 0.1250 mi and 1 fortnight = 14 days. 1 day = 24 h.

EXECUTE: $(180,000 \text{ furlongs/fortnight}) \left(\frac{0.125 \text{ mi}}{1 \text{ furlong}} \right) \left(\frac{1 \text{ fortnight}}{14 \text{ days}} \right) \left(\frac{1 \text{ day}}{24 \text{ h}} \right) = 67 \text{ mi/h}$

EVALUATE: A furlong is less than a mile and a fortnight is many hours, so the speed limit in mph is a much smaller number.

1.9. IDENTIFY: Convert miles/gallon to km/L.

SET UP: 1 mi = 1.609 km. 1 gallon = 3.788 L.

EXECUTE: (a) $55.0 \text{ miles/gallon} = (55.0 \text{ miles/gallon}) \left(\frac{1.609 \text{ km}}{1 \text{ mi}} \right) \left(\frac{1 \text{ gallon}}{3.788 \text{ L}} \right) = 23.4 \text{ km/L}$.

(b) The volume of gas required is $\frac{1500 \text{ km}}{23.4 \text{ km/L}} = 64.1 \text{ L}$. $\frac{64.1 \text{ L}}{45 \text{ L/tank}} = 1.4 \text{ tanks}$.

EVALUATE: 1 mi/gal = 0.425 km/L. A km is very roughly half a mile and there are roughly 4 liters in a gallon, so 1 mi/gal $\sim \frac{2}{4}$ km/L, which is roughly our result.

1.10. IDENTIFY: Convert units.

SET UP: Use the unit conversions given in the problem. Also, 100 cm = 1 m and 1000 g = 1 kg.

EXECUTE: (a) $\left(60 \frac{\text{mi}}{\text{h}} \right) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) \left(\frac{5280 \text{ ft}}{1 \text{ mi}} \right) = 88 \frac{\text{ft}}{\text{s}}$

(b) $\left(32 \frac{\text{ft}}{\text{s}^2} \right) \left(\frac{30.48 \text{ cm}}{1 \text{ ft}} \right) \left(\frac{1 \text{ m}}{100 \text{ cm}} \right) = 9.8 \frac{\text{m}}{\text{s}^2}$

(c) $\left(1.0 \frac{\text{g}}{\text{cm}^3} \right) \left(\frac{100 \text{ cm}}{1 \text{ m}} \right)^3 \left(\frac{1 \text{ kg}}{1000 \text{ g}} \right) = 10^3 \frac{\text{kg}}{\text{m}^3}$

EVALUATE: The relations $60 \text{ mi/h} = 88 \text{ ft/s}$ and $1 \text{ g/cm}^3 = 10^3 \text{ kg/m}^3$ are exact. The relation $32 \text{ ft/s}^2 = 9.8 \text{ m/s}^2$ is accurate to only two significant figures.

1.11. IDENTIFY: We know the density and mass; thus we can find the volume using the relation density = mass/volume = m/V . The radius is then found from the volume equation for a sphere and the result for the volume.

SET UP: Density = 19.5 g/cm^3 and $m_{\text{critical}} = 60.0 \text{ kg}$. For a sphere $V = \frac{4}{3} \pi r^3$.

EXECUTE: $V = m_{\text{critical}}/\text{density} = \left(\frac{60.0 \text{ kg}}{19.5 \text{ g/cm}^3} \right) \left(\frac{1000 \text{ g}}{1.0 \text{ kg}} \right) = 3080 \text{ cm}^3$.

$r = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3}{4\pi}(3080 \text{ cm}^3)} = 9.0 \text{ cm}$.

EVALUATE: The density is very large, so the 130-pound sphere is small in size.

1.12. IDENTIFY: Convert units.

SET UP: We know the equalities $1 \text{ mg} = 10^{-3} \text{ g}$, $1 \text{ } \mu\text{g} = 10^{-6} \text{ g}$, and $1 \text{ kg} = 10^3 \text{ g}$.

EXECUTE: (a) $(410 \text{ mg/day}) \left(\frac{10^{-3} \text{ g}}{1 \text{ mg}} \right) \left(\frac{1 \text{ } \mu\text{g}}{10^{-6} \text{ g}} \right) = 4.10 \times 10^5 \text{ } \mu\text{g/day}$.

(b) $(12 \text{ mg/kg})(75 \text{ kg}) = (900 \text{ mg}) \left(\frac{10^{-3} \text{ g}}{1 \text{ mg}} \right) = 0.900 \text{ g}$.

(c) The mass of each tablet is $(2.0 \text{ mg}) \left(\frac{10^{-3} \text{ g}}{1 \text{ mg}} \right) = 2.0 \times 10^{-3} \text{ g/day}$. The number of tablets required each

day is the number of grams recommended per day divided by the number of grams per tablet:

$$\frac{0.0030 \text{ g/day}}{2.0 \times 10^{-3} \text{ g/tablet}} = 1.5 \text{ tablet/day. Take 2 tablets each day.}$$

(d) $(0.000070 \text{ g/day}) \left(\frac{1 \text{ mg}}{10^{-3} \text{ g}} \right) = 0.070 \text{ mg/day}$.

EVALUATE: Quantities in medicine and nutrition are frequently expressed in a wide variety of units.

1.13. IDENTIFY: The percent error is the error divided by the quantity.

SET UP: The distance from Berlin to Paris is given to the nearest 10 km.

EXECUTE: (a) $\frac{10 \text{ m}}{890 \times 10^3 \text{ m}} = 1.1 \times 10^{-3} \%$.

(b) Since the distance was given as 890 km, the total distance should be 890,000 meters. We know the total distance to only three significant figures.

EVALUATE: In this case a very small percentage error has disastrous consequences.

1.14. IDENTIFY: When numbers are multiplied or divided, the number of significant figures in the result can be no greater than in the factor with the fewest significant figures. When we add or subtract numbers it is the location of the decimal that matters.

SET UP: 12 mm has two significant figures and 5.98 mm has three significant figures.

EXECUTE: (a) $(12 \text{ mm}) \times (5.98 \text{ mm}) = 72 \text{ mm}^2$ (two significant figures)

(b) $\frac{5.98 \text{ mm}}{12 \text{ mm}} = 0.50$ (also two significant figures)

(c) 36 mm (to the nearest millimeter)

(d) 6 mm

(e) 2.0 (two significant figures)

EVALUATE: The length of the rectangle is known only to the nearest mm, so the answers in parts (c) and (d) are known only to the nearest mm.

1.15. IDENTIFY: Use your calculator to display $\pi \times 10^7$. Compare that number to the number of seconds in a year.

SET UP: 1 yr = 365.24 days, 1 day = 24 h, and 1 h = 3600 s.

EXECUTE: $(365.24 \text{ days/yr}) \left(\frac{24 \text{ h}}{1 \text{ day}} \right) \left(\frac{3600 \text{ s}}{1 \text{ h}} \right) = 3.15567 \dots \times 10^7 \text{ s}$; $\pi \times 10^7 \text{ s} = 3.14159 \dots \times 10^7 \text{ s}$

The approximate expression is accurate to two significant figures. The percent error is 0.45%.

EVALUATE: The close agreement is a numerical accident.

1.16. IDENTIFY: Estimate the number of people and then use the estimates given in the problem to calculate the number of gallons.

SET UP: Estimate 3×10^8 people, so 2×10^8 cars.

EXECUTE: (Number of cars \times miles/car day)/(mi/gal) = gallons/day

$$(2 \times 10^8 \text{ cars} \times 10000 \text{ mi/yr/car} \times 1 \text{ yr}/365 \text{ days}) / (20 \text{ mi/gal}) = 3 \times 10^8 \text{ gal/day}$$

EVALUATE: The number of gallons of gas used each day approximately equals the population of the U.S.

1.17. IDENTIFY: Express 200 kg in pounds. Express each of 200 m, 200 cm and 200 mm in inches. Express 200 months in years.

SET UP: A mass of 1 kg is equivalent to a weight of about 2.2 lbs. 1 in. = 2.54 cm. 1 y = 12 months.

EXECUTE: (a) 200 kg is a weight of 440 lb. This is much larger than the typical weight of a man.

(b) $200 \text{ m} = (2.00 \times 10^4 \text{ cm}) \left(\frac{1 \text{ in.}}{2.54 \text{ cm}} \right) = 7.9 \times 10^3 \text{ inches}$. This is much greater than the height of a person.

(c) $200 \text{ cm} = 2.00 \text{ m} = 79 \text{ inches} = 6.6 \text{ ft}$. Some people are this tall, but not an ordinary man.

(d) $200 \text{ mm} = 0.200 \text{ m} = 7.9 \text{ inches}$. This is much too short.

(e) $200 \text{ months} = (200 \text{ mon}) \left(\frac{1 \text{ y}}{12 \text{ mon}} \right) = 17 \text{ y}$. This is the age of a teenager; a middle-aged man is much older than this.

EVALUATE: None are plausible. When specifying the value of a measured quantity it is essential to give the units in which it is being expressed.

1.18. IDENTIFY: The number of kernels can be calculated as $N = V_{\text{bottle}}/V_{\text{kernel}}$.

SET UP: Based on an Internet search, Iowa corn farmers use a sieve having a hole size of 0.3125 in. \cong 8 mm to remove kernel fragments. Therefore estimate the average kernel length as 10 mm, the width as 6 mm and the depth as 3 mm. We must also apply the conversion factors $1 \text{ L} = 1000 \text{ cm}^3$ and $1 \text{ cm} = 10 \text{ mm}$.

EXECUTE: The volume of the kernel is: $V_{\text{kernel}} = (10 \text{ mm})(6 \text{ mm})(3 \text{ mm}) = 180 \text{ mm}^3$. The bottle's volume is: $V_{\text{bottle}} = (2.0 \text{ L})[(1000 \text{ cm}^3)/(1.0 \text{ L})][(10 \text{ mm})^3/(1.0 \text{ cm})^3] = 2.0 \times 10^6 \text{ mm}^3$. The number of kernels is then $N_{\text{kernels}} = V_{\text{bottle}}/V_{\text{kernels}} \approx (2.0 \times 10^6 \text{ mm}^3)/(180 \text{ mm}^3) = 11,000 \text{ kernels}$.

EVALUATE: This estimate is highly dependent upon your estimate of the kernel dimensions. And since these dimensions vary amongst the different available types of corn, acceptable answers could range from 6,500 to 20,000.

1.19. IDENTIFY: Estimate the number of pages and the number of words per page.

SET UP: Assuming the two-volume edition, there are approximately a thousand pages, and each page has between 500 and a thousand words (counting captions and the smaller print, such as the end-of-chapter exercises and problems).

EXECUTE: An estimate for the number of words is about 10^6 .

EVALUATE: We can expect that this estimate is accurate to within a factor of 10.

1.20. IDENTIFY: Approximate the number of breaths per minute. Convert minutes to years and cm^3 to m^3 to find the volume in m^3 breathed in a year.

SET UP: Assume 10 breaths/min. $1 \text{ y} = (365 \text{ d}) \left(\frac{24 \text{ h}}{1 \text{ d}} \right) \left(\frac{60 \text{ min}}{1 \text{ h}} \right) = 5.3 \times 10^5 \text{ min}$. $10^2 \text{ cm} = 1 \text{ m}$ so

$10^6 \text{ cm}^3 = 1 \text{ m}^3$. The volume of a sphere is $V = \frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$, where r is the radius and d is the diameter.

Don't forget to account for four astronauts.

EXECUTE: (a) The volume is $(4)(10 \text{ breaths/min})(500 \times 10^{-6} \text{ m}^3) \left(\frac{5.3 \times 10^5 \text{ min}}{1 \text{ y}} \right) = 1 \times 10^4 \text{ m}^3/\text{yr}$.

$$(b) d = \left(\frac{6V}{\pi} \right)^{1/3} = \left(\frac{6[1 \times 10^4 \text{ m}^3]}{\pi} \right)^{1/3} = 27 \text{ m}$$

EVALUATE: Our estimate assumes that each cm^3 of air is breathed in only once, where in reality not all the oxygen is absorbed from the air in each breath. Therefore, a somewhat smaller volume would actually be required.

1.21. IDENTIFY: Estimate the number of blinks per minute. Convert minutes to years. Estimate the typical lifetime in years.

SET UP: Estimate that we blink 10 times per minute. $1 \text{ y} = 365 \text{ days}$. $1 \text{ day} = 24 \text{ h}$, $1 \text{ h} = 60 \text{ min}$. Use 80 years for the lifetime.

EXECUTE: The number of blinks is $(10 \text{ per min}) \left(\frac{60 \text{ min}}{1 \text{ h}} \right) \left(\frac{24 \text{ h}}{1 \text{ day}} \right) \left(\frac{365 \text{ days}}{1 \text{ y}} \right) (80 \text{ y/lifetime}) = 4 \times 10^8$

EVALUATE: Our estimate of the number of blinks per minute can be off by a factor of two but our calculation is surely accurate to a power of 10.

- 1.22. IDENTIFY:** Estimate the number of beats per minute and the duration of a lifetime. The volume of blood pumped during this interval is then the volume per beat multiplied by the total beats.

SET UP: An average middle-aged (40 year-old) adult at rest has a heart rate of roughly 75 beats per minute. To calculate the number of beats in a lifetime, use the current average lifespan of 80 years.

EXECUTE: $N_{\text{beats}} = (75 \text{ beats/min}) \left(\frac{60 \text{ min}}{1 \text{ h}} \right) \left(\frac{24 \text{ h}}{1 \text{ day}} \right) \left(\frac{365 \text{ days}}{\text{yr}} \right) \left(\frac{80 \text{ yr}}{\text{lifespan}} \right) = 3 \times 10^9 \text{ beats/lifespan}$

$V_{\text{blood}} = (50 \text{ cm}^3/\text{beat}) \left(\frac{1 \text{ L}}{1000 \text{ cm}^3} \right) \left(\frac{1 \text{ gal}}{3.788 \text{ L}} \right) \left(\frac{3 \times 10^9 \text{ beats}}{\text{lifespan}} \right) = 4 \times 10^7 \text{ gal/lifespan}$

EVALUATE: This is a very large volume.

- 1.23. IDENTIFY:** Estimation problem

SET UP: Estimate that the pile is 18 in. \times 18 in. \times 5 ft 8 in.. Use the density of gold to calculate the mass of gold in the pile and from this calculate the dollar value.

EXECUTE: The volume of gold in the pile is $V = 18 \text{ in.} \times 18 \text{ in.} \times 68 \text{ in.} = 22,000 \text{ in.}^3$. Convert to cm^3 :

$$V = 22,000 \text{ in.}^3 (1000 \text{ cm}^3/61.02 \text{ in.}^3) = 3.6 \times 10^5 \text{ cm}^3.$$

The density of gold is 19.3 g/cm^3 , so the mass of this volume of gold is

$$m = (19.3 \text{ g/cm}^3)(3.6 \times 10^5 \text{ cm}^3) = 7 \times 10^6 \text{ g.}$$

The monetary value of one gram is \$10, so the gold has a value of $(\$10/\text{gram})(7 \times 10^6 \text{ grams}) = \7×10^7 , or about $\$100 \times 10^6$ (one hundred million dollars).

EVALUATE: This is quite a large pile of gold, so such a large monetary value is reasonable.

- 1.24. IDENTIFY:** Estimate the diameter of a drop and from that calculate the volume of a drop, in m^3 . Convert m^3 to L.

SET UP: Estimate the diameter of a drop to be $d = 2 \text{ mm}$. The volume of a spherical drop is

$$V = \frac{4}{3} \pi r^3 = \frac{1}{6} \pi d^3. 10^3 \text{ cm}^3 = 1 \text{ L.}$$

EXECUTE: $V = \frac{1}{6} \pi (0.2 \text{ cm})^3 = 4 \times 10^{-3} \text{ cm}^3$. The number of drops in 1.0 L is $\frac{1000 \text{ cm}^3}{4 \times 10^{-3} \text{ cm}^3} = 2 \times 10^5$

EVALUATE: Since $V \sim d^3$, if our estimate of the diameter of a drop is off by a factor of 2 then our estimate of the number of drops is off by a factor of 8.

- 1.25. IDENTIFY:** Estimate the number of students and the average number of pizzas eaten by each student in a school year.

SET UP: Assume a school of a thousand students, each of whom averages ten pizzas a year (perhaps an underestimate)

EXECUTE: They eat a total of 10^4 pizzas.

EVALUATE: The same answer applies to a school of 250 students averaging 40 pizzas a year each.

- 1.26. IDENTIFY:** The displacements must be added as vectors and the magnitude of the sum depends on the relative orientation of the two displacements.

SET UP: The sum with the largest magnitude is when the two displacements are parallel and the sum with the smallest magnitude is when the two displacements are antiparallel.

EXECUTE: The orientations of the displacements that give the desired sum are shown in Figure 1.26.

EVALUATE: The orientations of the two displacements can be chosen such that the sum has any value between 0.6 m and 4.2 m.

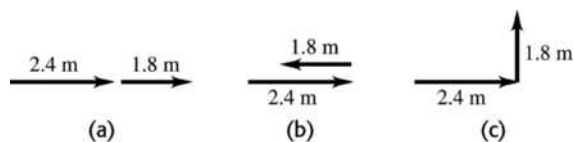


Figure 1.26

- 1.27. **IDENTIFY:** Draw each subsequent displacement tail to head with the previous displacement. The resultant displacement is the single vector that points from the starting point to the stopping point.

SET UP: Call the three displacements \vec{A} , \vec{B} , and \vec{C} . The resultant displacement \vec{R} is given by $\vec{R} = \vec{A} + \vec{B} + \vec{C}$.

EXECUTE: The vector addition diagram is given in Figure 1.27. Careful measurement gives that \vec{R} is 7.8 km, 38° north of east.

EVALUATE: The magnitude of the resultant displacement, 7.8 km, is less than the sum of the magnitudes of the individual displacements, 2.6 km + 4.0 km + 3.1 km.

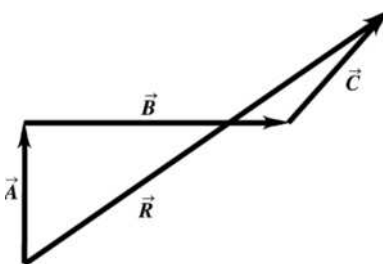


Figure 1.27

- 1.28. **IDENTIFY:** Draw the vector addition diagram to scale.

SET UP: The two vectors \vec{A} and \vec{B} are specified in the figure that accompanies the problem.

EXECUTE: (a) The diagram for $\vec{C} = \vec{A} + \vec{B}$ is given in Figure 1.28a. Measuring the length and angle of \vec{C} gives $C = 9.0$ m and an angle of $\theta = 34^\circ$.

(b) The diagram for $\vec{D} = \vec{A} - \vec{B}$ is given in Figure 1.28b. Measuring the length and angle of \vec{D} gives $D = 22$ m and an angle of $\theta = 250^\circ$.

(c) $-\vec{A} - \vec{B} = -(\vec{A} + \vec{B})$, so $-\vec{A} - \vec{B}$ has a magnitude of 9.0 m (the same as $\vec{A} + \vec{B}$) and an angle with the $+x$ axis of 214° (opposite to the direction of $\vec{A} + \vec{B}$).

(d) $\vec{B} - \vec{A} = -(\vec{A} - \vec{B})$, so $\vec{B} - \vec{A}$ has a magnitude of 22 m and an angle with the $+x$ axis of 70° (opposite to the direction of $\vec{A} - \vec{B}$).

EVALUATE: The vector $-\vec{A}$ is equal in magnitude and opposite in direction to the vector \vec{A} .

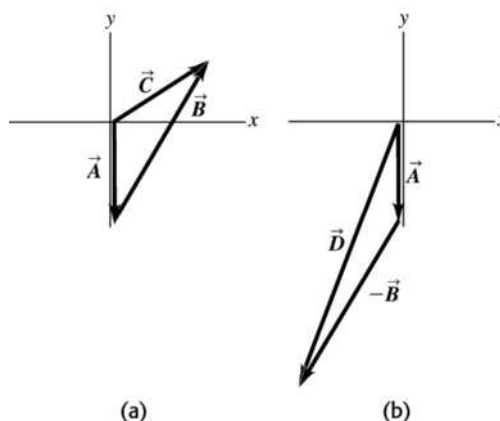


Figure 1.28

- 1.29. IDENTIFY:** Since she returns to the starting point, the vector sum of the four displacements must be zero.
- SET UP:** Call the three given displacements \vec{A} , \vec{B} , and \vec{C} , and call the fourth displacement \vec{D} .
- $$\vec{A} + \vec{B} + \vec{C} + \vec{D} = 0.$$
- EXECUTE:** The vector addition diagram is sketched in Figure 1.29. Careful measurement gives that \vec{D} is 144 m, 41° south of west.
- EVALUATE:** \vec{D} is equal in magnitude and opposite in direction to the sum $\vec{A} + \vec{B} + \vec{C}$.

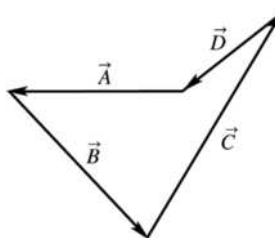


Figure 1.29

- 1.30. IDENTIFY:** $\tan \theta = \frac{A_y}{A_x}$, for θ measured counterclockwise from the $+x$ -axis.
- SET UP:** A sketch of A_x , A_y and \vec{A} tells us the quadrant in which \vec{A} lies.
- EXECUTE:**
- (a) $\tan \theta = \frac{A_y}{A_x} = \frac{-1.00 \text{ m}}{2.00 \text{ m}} = -0.500$. $\theta = \tan^{-1}(-0.500) = 360^\circ - 26.6^\circ = 333^\circ$.
- (b) $\tan \theta = \frac{A_y}{A_x} = \frac{1.00 \text{ m}}{2.00 \text{ m}} = 0.500$. $\theta = \tan^{-1}(0.500) = 26.6^\circ$.
- (c) $\tan \theta = \frac{A_y}{A_x} = \frac{1.00 \text{ m}}{-2.00 \text{ m}} = -0.500$. $\theta = \tan^{-1}(-0.500) = 180^\circ - 26.6^\circ = 153^\circ$.
- (d) $\tan \theta = \frac{A_y}{A_x} = \frac{-1.00 \text{ m}}{-2.00 \text{ m}} = 0.500$. $\theta = \tan^{-1}(0.500) = 180^\circ + 26.6^\circ = 207^\circ$
- EVALUATE:** The angles 26.6° and 207° have the same tangent. Our sketch tells us which is the correct value of θ .
- 1.31. IDENTIFY:** For each vector \vec{V} , use that $V_x = V \cos \theta$ and $V_y = V \sin \theta$, when θ is the angle \vec{V} makes with the $+x$ axis, measured counterclockwise from the axis.

SET UP: For \vec{A} , $\theta = 270.0^\circ$. For \vec{B} , $\theta = 60.0^\circ$. For \vec{C} , $\theta = 205.0^\circ$. For \vec{D} , $\theta = 143.0^\circ$.

EXECUTE: $A_x = 0$, $A_y = -8.00$ m. $B_x = 7.50$ m, $B_y = 13.0$ m. $C_x = -10.9$ m, $C_y = -5.07$ m.

$D_x = -7.99$ m, $D_y = 6.02$ m.

EVALUATE: The signs of the components correspond to the quadrant in which the vector lies.

- 1.32. IDENTIFY:** Given the direction and one component of a vector, find the other component and the magnitude.

SET UP: Use the tangent of the given angle and the definition of vector magnitude.

EXECUTE: (a) $\tan 34.0^\circ = \frac{|A_x|}{|A_y|}$

$$|A_y| = \frac{|A_x|}{\tan 34.0^\circ} = \frac{16.0 \text{ m}}{\tan 34.0^\circ} = 23.72 \text{ m}$$

$A_y = -23.7$ m.

(b) $A = \sqrt{A_x^2 + A_y^2} = 28.6$ m.

EVALUATE: The magnitude is greater than either of the components.

- 1.33. IDENTIFY:** Given the direction and one component of a vector, find the other component and the magnitude.

SET UP: Use the tangent of the given angle and the definition of vector magnitude.

EXECUTE: (a) $\tan 32.0^\circ = \frac{|A_x|}{|A_y|}$

$$|A_x| = (13.0 \text{ m})\tan 32.0^\circ = 8.12 \text{ m. } A_x = -8.12 \text{ m.}$$

(b) $A = \sqrt{A_x^2 + A_y^2} = 15.3$ m.

EVALUATE: The magnitude is greater than either of the components.

- 1.34. IDENTIFY:** Find the vector sum of the three given displacements.

SET UP: Use coordinates for which $+x$ is east and $+y$ is north. The driver's vector displacements are:

$\vec{A} = 2.6$ km, 0° of north; $\vec{B} = 4.0$ km, 0° of east; $\vec{C} = 3.1$ km, 45° north of east.

EXECUTE: $R_x = A_x + B_x + C_x = 0 + 4.0 \text{ km} + (3.1 \text{ km})\cos(45^\circ) = 6.2$ km; $R_y = A_y + B_y + C_y =$

$2.6 \text{ km} + 0 + (3.1 \text{ km})(\sin 45^\circ) = 4.8$ km; $R = \sqrt{R_x^2 + R_y^2} = 7.8$ km; $\theta = \tan^{-1}[(4.8 \text{ km})/(6.2 \text{ km})] = 38^\circ$;

$\vec{R} = 7.8$ km, 38° north of east. This result is confirmed by the sketch in Figure 1.34.

EVALUATE: Both R_x and R_y are positive and \vec{R} is in the first quadrant.

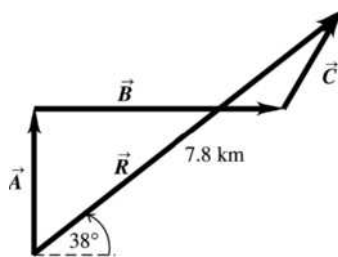


Figure 1.34

- 1.35. IDENTIFY:** If $\vec{C} = \vec{A} + \vec{B}$, then $C_x = A_x + B_x$ and $C_y = A_y + B_y$. Use C_x and C_y to find the magnitude and direction of \vec{C} .

SET UP: From Figure E1.28 in the textbook, $A_x = 0$, $A_y = -8.00$ m and $B_x = +B \sin 30.0^\circ = 7.50$ m,

$B_y = +B \cos 30.0^\circ = 13.0$ m.

EXECUTE: (a) $\vec{C} = \vec{A} + \vec{B}$ so $C_x = A_x + B_x = 7.50$ m and $C_y = A_y + B_y = +5.00$ m. $C = 9.01$ m.

$$\tan \theta = \frac{C_y}{C_x} = \frac{5.00 \text{ m}}{7.50 \text{ m}} \text{ and } \theta = 33.7^\circ.$$

(b) $\vec{B} + \vec{A} = \vec{A} + \vec{B}$, so $\vec{B} + \vec{A}$ has magnitude 9.01 m and direction specified by 33.7° .

(c) $\vec{D} = \vec{A} - \vec{B}$ so $D_x = A_x - B_x = -7.50$ m and $D_y = A_y - B_y = -21.0$ m. $D = 22.3$ m.

$$\tan \phi = \frac{D_y}{D_x} = \frac{-21.0 \text{ m}}{-7.50 \text{ m}} \text{ and } \phi = 70.3^\circ. \vec{D} \text{ is in the 3}^{\text{rd}} \text{ quadrant and the angle } \theta \text{ counterclockwise from the}$$

$+x$ axis is $180^\circ + 70.3^\circ = 250.3^\circ$.

(d) $\vec{B} - \vec{A} = -(\vec{A} - \vec{B})$, so $\vec{B} - \vec{A}$ has magnitude 22.3 m and direction specified by $\theta = 70.3^\circ$.

EVALUATE: These results agree with those calculated from a scale drawing in Problem 1.28.

1.36. IDENTIFY: Use Equations (1.7) and (1.8) to calculate the magnitude and direction of each of the given vectors.

SET UP: A sketch of A_x , A_y , and \vec{A} tells us the quadrant in which \vec{A} lies.

EXECUTE: (a) $\sqrt{(-8.60 \text{ cm})^2 + (5.20 \text{ cm})^2} = 10.0$ cm, $\arctan\left(\frac{5.20}{-8.60}\right) = 148.8^\circ$ (which is $180^\circ - 31.2^\circ$).

(b) $\sqrt{(-9.7 \text{ m})^2 + (-2.45 \text{ m})^2} = 10.0$ m, $\arctan\left(\frac{-2.45}{-9.7}\right) = 14^\circ + 180^\circ = 194^\circ$.

(c) $\sqrt{(7.75 \text{ km})^2 + (-2.70 \text{ km})^2} = 8.21$ km, $\arctan\left(\frac{-2.7}{7.75}\right) = 340.8^\circ$ (which is $360^\circ - 19.2^\circ$).

EVALUATE: In each case the angle is measured counterclockwise from the $+x$ axis. Our results for θ agree with our sketches.

1.37. IDENTIFY: Vector addition problem. We are given the magnitude and direction of three vectors and are asked to find their sum.

SET UP:

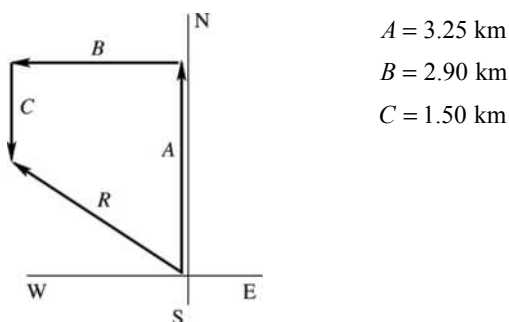
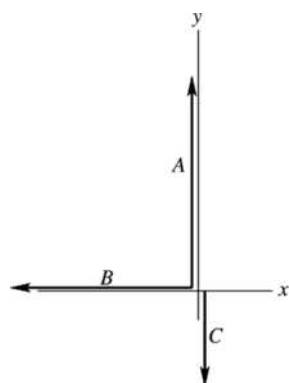


Figure 1.37a

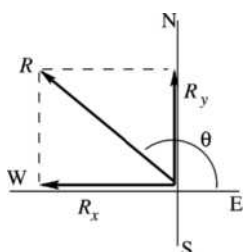
Select a coordinate system where $+x$ is east and $+y$ is north. Let \vec{A} , \vec{B} and \vec{C} be the three displacements of the professor. Then the resultant displacement \vec{R} is given by $\vec{R} = \vec{A} + \vec{B} + \vec{C}$. By the method of components, $R_x = A_x + B_x + C_x$ and $R_y = A_y + B_y + C_y$. Find the x and y components of each vector; add them to find the components of the resultant. Then the magnitude and direction of the resultant can be found from its x and y components that we have calculated. As always it is essential to draw a sketch.

EXECUTE:



$$\begin{aligned}
 A_x &= 0, A_y = +3.25 \text{ km} \\
 B_x &= -2.90 \text{ km}, B_y = 0 \\
 C_x &= 0, C_y = -1.50 \text{ km} \\
 R_x &= A_x + B_x + C_x \\
 R_x &= 0 - 2.90 \text{ km} + 0 = -2.90 \text{ km} \\
 R_y &= A_y + B_y + C_y \\
 R_y &= 3.25 \text{ km} + 0 - 1.50 \text{ km} = 1.75 \text{ km}
 \end{aligned}$$

Figure 1.37b



$$\begin{aligned}
 R &= \sqrt{R_x^2 + R_y^2} = \sqrt{(-2.90 \text{ km})^2 + (1.75 \text{ km})^2} \\
 R &= 3.39 \text{ km} \\
 \tan \theta &= \frac{R_y}{R_x} = \frac{1.75 \text{ km}}{-2.90 \text{ km}} = -0.603 \\
 \theta &= 148.9^\circ
 \end{aligned}$$

Figure 1.37c

The angle θ measured counterclockwise from the $+x$ -axis. In terms of compass directions, the resultant displacement is 31.1° N of W.

EVALUATE: $R_x < 0$ and $R_y > 0$, so \vec{R} is in 2nd quadrant. This agrees with the vector addition diagram.

1.38. IDENTIFY: We know the vector sum and want to find the magnitude of the vectors. Use the method of components.

SET UP: The two vectors \vec{A} and \vec{B} and their resultant \vec{C} are shown in Figure 1.38. Let $+y$ be in the direction of the resultant. $A = B$.

EXECUTE: $C_y = A_y + B_y$. $372 \text{ N} = 2A \cos 43.0^\circ$ and $A = 254 \text{ N}$.

EVALUATE: The sum of the magnitudes of the two forces exceeds the magnitude of the resultant force because only a component of each force is upward.

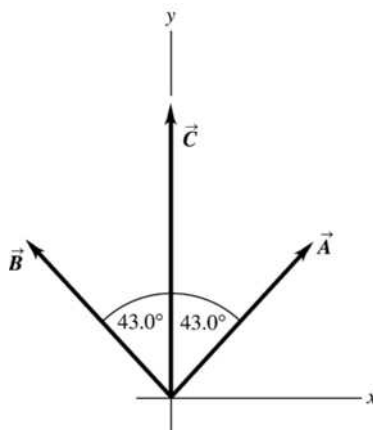
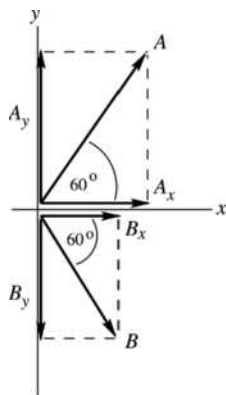


Figure 1.38

1.39. IDENTIFY: Vector addition problem. $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$.

SET UP: Find the x - and y -components of \vec{A} and \vec{B} . Then the x - and y -components of the vector sum are calculated from the x - and y -components of \vec{A} and \vec{B} .

EXECUTE:



$$A_x = A \cos(60.0^\circ)$$

$$A_x = (2.80 \text{ cm}) \cos(60.0^\circ) = +1.40 \text{ cm}$$

$$A_y = A \sin(60.0^\circ)$$

$$A_y = (2.80 \text{ cm}) \sin(60.0^\circ) = +2.425 \text{ cm}$$

$$B_x = B \cos(-60.0^\circ)$$

$$B_x = (1.90 \text{ cm}) \cos(-60.0^\circ) = +0.95 \text{ cm}$$

$$B_y = B \sin(-60.0^\circ)$$

$$B_y = (1.90 \text{ cm}) \sin(-60.0^\circ) = -1.645 \text{ cm}$$

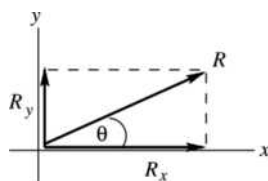
Note that the signs of the components correspond to the directions of the component vectors.

Figure 1.39a

(a) Now let $\vec{R} = \vec{A} + \vec{B}$.

$$R_x = A_x + B_x = +1.40 \text{ cm} + 0.95 \text{ cm} = +2.35 \text{ cm}.$$

$$R_y = A_y + B_y = +2.425 \text{ cm} - 1.645 \text{ cm} = +0.78 \text{ cm}.$$



$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(2.35 \text{ cm})^2 + (0.78 \text{ cm})^2}$$

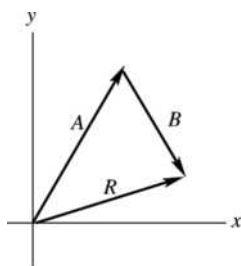
$$R = 2.48 \text{ cm}$$

$$\tan \theta = \frac{R_y}{R_x} = \frac{+0.78 \text{ cm}}{+2.35 \text{ cm}} = +0.3319$$

$$\theta = 18.4^\circ$$

Figure 1.39b

EVALUATE: The vector addition diagram for $\vec{R} = \vec{A} + \vec{B}$ is



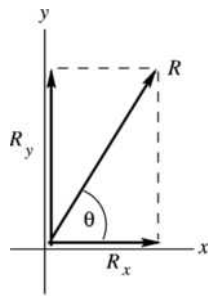
\vec{R} is in the 1st quadrant, with $|R_y| < |R_x|$, in agreement with our calculation.

Figure 1.39c

(b) EXECUTE: Now let $\vec{R} = \vec{A} - \vec{B}$.

$$R_x = A_x - B_x = +1.40 \text{ cm} - 0.95 \text{ cm} = +0.45 \text{ cm}.$$

$$R_y = A_y - B_y = +2.425 \text{ cm} + 1.645 \text{ cm} = +4.070 \text{ cm}.$$



$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(0.45 \text{ cm})^2 + (4.070 \text{ cm})^2}$$

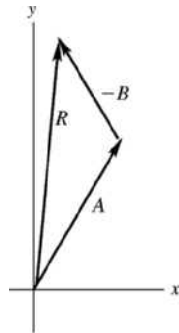
$$R = 4.09 \text{ cm}$$

$$\tan \theta = \frac{R_y}{R_x} = \frac{4.070 \text{ cm}}{0.45 \text{ cm}} = +9.044$$

$$\theta = 83.7^\circ$$

Figure 1.39d

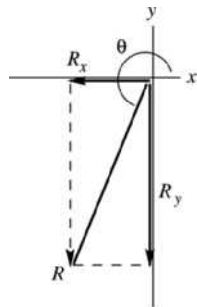
EVALUATE: The vector addition diagram for $\vec{R} = \vec{A} + (-\vec{B})$ is



\vec{R} is in the 1st quadrant, with $|R_x| < |R_y|$, in agreement with our calculation.

Figure 1.39e

(c) EXECUTE:



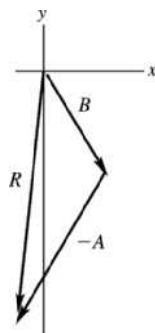
$$\vec{B} - \vec{A} = -(\vec{A} - \vec{B})$$

$\vec{B} - \vec{A}$ and $\vec{A} - \vec{B}$ are equal in magnitude and opposite in direction.

$$R = 4.09 \text{ cm} \text{ and } \theta = 83.7^\circ + 180^\circ = 264^\circ$$

Figure 1.39f

EVALUATE: The vector addition diagram for $\vec{R} = \vec{B} + (-\vec{A})$ is



\vec{R} is in the 3rd quadrant, with $|R_x| < |R_y|$, in agreement with our calculation.

Figure 1.39g

1.40. IDENTIFY: The general expression for a vector written in terms of components and unit vectors is $\vec{A} = A_x \hat{i} + A_y \hat{j}$.

SET UP: $5.0\vec{B} = 5.0(4\hat{i} - 6\hat{j}) = 20\hat{i} - 30\hat{j}$

EXECUTE: (a) $A_x = 5.0$, $A_y = -6.3$ (b) $A_x = 11.2$, $A_y = -9.91$ (c) $A_x = -15.0$, $A_y = 22.4$

(d) $A_x = 20$, $A_y = -30$

EVALUATE: The components are signed scalars.

1.41. IDENTIFY: Find the components of each vector and then use Eq. (1.14).

SET UP: $A_x = 0$, $A_y = -8.00$ m. $B_x = 7.50$ m, $B_y = 13.0$ m. $C_x = -10.9$ m, $C_y = -5.07$ m.

$D_x = -7.99$ m, $D_y = 6.02$ m.

EXECUTE: $\vec{A} = (-8.00 \text{ m})\hat{j}$; $\vec{B} = (7.50 \text{ m})\hat{i} + (13.0 \text{ m})\hat{j}$; $\vec{C} = (-10.9 \text{ m})\hat{i} + (-5.07 \text{ m})\hat{j}$;

$\vec{D} = (-7.99 \text{ m})\hat{i} + (6.02 \text{ m})\hat{j}$.

EVALUATE: All these vectors lie in the xy -plane and have no z -component.

1.42. IDENTIFY: Find A and B . Find the vector difference using components.

SET UP: Deduce the x - and y -components and use Eq. (1.8).

EXECUTE: (a) $\vec{A} = 4.00\hat{i} + 7.00\hat{j}$; $A_x = +4.00$; $A_y = +7.00$.

$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(4.00)^2 + (7.00)^2} = 8.06$. $\vec{B} = 5.00\hat{i} - 2.00\hat{j}$; $B_x = +5.00$; $B_y = -2.00$;

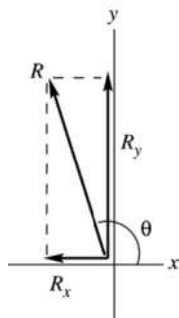
$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(5.00)^2 + (-2.00)^2} = 5.39$.

EVALUATE: Note that the magnitudes of \vec{A} and \vec{B} are each larger than either of their components.

EXECUTE: (b) $\vec{A} - \vec{B} = 4.00\hat{i} + 7.00\hat{j} - (5.00\hat{i} - 2.00\hat{j}) = (4.00 - 5.00)\hat{i} + (7.00 + 2.00)\hat{j}$.

$\vec{A} - \vec{B} = -1.00\hat{i} + 9.00\hat{j}$

(c) Let $\vec{R} = \vec{A} - \vec{B} = -1.00\hat{i} + 9.00\hat{j}$. Then $R_x = -1.00$, $R_y = 9.00$.



$$R = \sqrt{R_x^2 + R_y^2}$$

$$R = \sqrt{(-1.00)^2 + (9.00)^2} = 9.06$$

$$\tan \theta = \frac{R_y}{R_x} = \frac{9.00}{-1.00} = -9.00$$

$$\theta = -83.6^\circ + 180^\circ = 96.3^\circ$$

Figure 1.42

EVALUATE: $R_x < 0$ and $R_y > 0$, so \vec{R} is in the 2nd quadrant.

1.43. IDENTIFY: Use trig to find the components of each vector. Use Eq. (1.11) to find the components of the vector sum. Eq. (1.14) expresses a vector in terms of its components.

SET UP: Use the coordinates in the figure that accompanies the problem.

EXECUTE: (a) $\vec{A} = (3.60 \text{ m})\cos 70.0^\circ \hat{i} + (3.60 \text{ m})\sin 70.0^\circ \hat{j} = (1.23 \text{ m})\hat{i} + (3.38 \text{ m})\hat{j}$

$\vec{B} = -(2.40 \text{ m})\cos 30.0^\circ \hat{i} - (2.40 \text{ m})\sin 30.0^\circ \hat{j} = (-2.08 \text{ m})\hat{i} + (-1.20 \text{ m})\hat{j}$

(b) $\vec{C} = (3.00)\vec{A} - (4.00)\vec{B} = (3.00)(1.23 \text{ m})\hat{i} + (3.00)(3.38 \text{ m})\hat{j} - (4.00)(-2.08 \text{ m})\hat{i} - (4.00)(-1.20 \text{ m})\hat{j}$
 $= (12.01 \text{ m})\hat{i} + (14.94)\hat{j}$

(c) From Equations (1.7) and (1.8),

$$C = \sqrt{(12.01 \text{ m})^2 + (14.94 \text{ m})^2} = 19.17 \text{ m}, \arctan\left(\frac{14.94 \text{ m}}{12.01 \text{ m}}\right) = 51.2^\circ$$

EVALUATE: C_x and C_y are both positive, so θ is in the first quadrant.

1.44. IDENTIFY: A unit vector has magnitude equal to 1.

SET UP: The magnitude of a vector is given in terms of its components by Eq. (1.12).

EXECUTE: (a) $|\hat{i} + \hat{j} + \hat{k}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3} \neq 1$ so it is not a unit vector.

(b) $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$. If any component is greater than +1 or less than -1, $|\vec{A}| > 1$, so it cannot be a unit vector. \vec{A} can have negative components since the minus sign goes away when the component is squared.

(c) $|\vec{A}| = 1$ gives $\sqrt{a^2(3.0)^2 + a^2(4.0)^2} = 1$ and $\sqrt{a^2}\sqrt{25} = 1$. $a = \pm\frac{1}{5.0} = \pm 0.20$.

EVALUATE: The magnitude of a vector is greater than the magnitude of any of its components.

1.45. IDENTIFY: $\vec{A} \cdot \vec{B} = AB \cos \phi$

SET UP: For \vec{A} and \vec{B} , $\phi = 150.0^\circ$. For \vec{B} and \vec{C} , $\phi = 145.0^\circ$. For \vec{A} and \vec{C} , $\phi = 65.0^\circ$.

EXECUTE: (a) $\vec{A} \cdot \vec{B} = (8.00 \text{ m})(15.0 \text{ m})\cos 150.0^\circ = -104 \text{ m}^2$

(b) $\vec{B} \cdot \vec{C} = (15.0 \text{ m})(12.0 \text{ m})\cos 145.0^\circ = -148 \text{ m}^2$

(c) $\vec{A} \cdot \vec{C} = (8.00 \text{ m})(12.0 \text{ m})\cos 65.0^\circ = 40.6 \text{ m}^2$

EVALUATE: When $\phi < 90^\circ$ the scalar product is positive and when $\phi > 90^\circ$ the scalar product is negative.

1.46. IDENTIFY: Target variables are $\vec{A} \cdot \vec{B}$ and the angle ϕ between the two vectors.

SET UP: We are given \vec{A} and \vec{B} in unit vector form and can take the scalar product using Eq. (1.19). The angle ϕ can then be found from Eq. (1.18).

EXECUTE: (a) $\vec{A} = 4.00\hat{i} + 7.00\hat{j}$, $\vec{B} = 5.00\hat{i} - 2.00\hat{j}$; $A = 8.06$, $B = 5.39$.

$\vec{A} \cdot \vec{B} = (4.00\hat{i} + 7.00\hat{j}) \cdot (5.00\hat{i} - 2.00\hat{j}) = (4.00)(5.00) + (7.00)(-2.00) = 20.0 - 14.0 = +6.00$.

(b) $\cos \phi = \frac{\vec{A} \cdot \vec{B}}{AB} = \frac{6.00}{(8.06)(5.39)} = 0.1382$; $\phi = 82.1^\circ$.

EVALUATE: The component of \vec{B} along \vec{A} is in the same direction as \vec{A} , so the scalar product is positive and the angle ϕ is less than 90° .

1.47. IDENTIFY: For all of these pairs of vectors, the angle is found from combining Eqs. (1.18) and (1.21),

to give the angle ϕ as $\phi = \arccos\left(\frac{\vec{A} \cdot \vec{B}}{AB}\right) = \arccos\left(\frac{A_x B_x + A_y B_y}{AB}\right)$.

SET UP: Eq. (1.14) shows how to obtain the components for a vector written in terms of unit vectors.

EXECUTE: (a) $\vec{A} \cdot \vec{B} = -22$, $A = \sqrt{40}$, $B = \sqrt{13}$, and so $\phi = \arccos\left(\frac{-22}{\sqrt{40}\sqrt{13}}\right) = 165^\circ$.

(b) $\vec{A} \cdot \vec{B} = 60$, $A = \sqrt{34}$, $B = \sqrt{136}$, $\phi = \arccos\left(\frac{60}{\sqrt{34}\sqrt{136}}\right) = 28^\circ$.

(c) $\vec{A} \cdot \vec{B} = 0$ and $\phi = 90^\circ$.

EVALUATE: If $\vec{A} \cdot \vec{B} > 0$, $0 \leq \phi < 90^\circ$. If $\vec{A} \cdot \vec{B} < 0$, $90^\circ < \phi \leq 180^\circ$. If $\vec{A} \cdot \vec{B} = 0$, $\phi = 90^\circ$ and the two vectors are perpendicular.

1.48. IDENTIFY: Target variable is the vector $\vec{A} \times \vec{B}$ expressed in terms of unit vectors.

SET UP: We are given \vec{A} and \vec{B} in unit vector form and can take the vector product using Eq. (1.24).

EXECUTE: $\vec{A} = 4.00\hat{i} + 7.00\hat{j}$, $\vec{B} = 5.00\hat{i} - 2.00\hat{j}$.

$\vec{A} \times \vec{B} = (4.00\hat{i} + 7.00\hat{j}) \times (5.00\hat{i} - 2.00\hat{j}) = 20.0\hat{i} \times \hat{i} - 8.00\hat{i} \times \hat{j} + 35.0\hat{j} \times \hat{i} - 14.0\hat{j} \times \hat{j}$. But $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = 0$ and $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{i} = -\hat{k}$, so $\vec{A} \times \vec{B} = -8.00\hat{k} + 35.0(-\hat{k}) = -43.0\hat{k}$. The magnitude of $\vec{A} \times \vec{B}$ is 43.0.

EVALUATE: Sketch the vectors \vec{A} and \vec{B} in a coordinate system where the xy -plane is in the plane of the paper and the z -axis is directed out toward you. By the right-hand rule $\vec{A} \times \vec{B}$ is directed into the plane of the paper, in the $-z$ -direction. This agrees with the above calculation that used unit vectors.

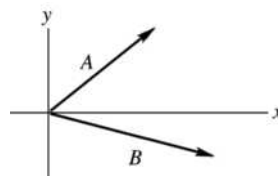


Figure 1.48

- 1.49. IDENTIFY:** $\vec{A} \times \vec{D}$ has magnitude $AD \sin \phi$. Its direction is given by the right-hand rule.
SET UP: $\phi = 180^\circ - 53^\circ = 127^\circ$
EXECUTE: (a) $|\vec{A} \times \vec{D}| = (8.00 \text{ m})(10.0 \text{ m}) \sin 127^\circ = 63.9 \text{ m}^2$. The right-hand rule says $\vec{A} \times \vec{D}$ is in the $-z$ -direction (into the page).
(b) $\vec{D} \times \vec{A}$ has the same magnitude as $\vec{A} \times \vec{D}$ and is in the opposite direction.
EVALUATE: The component of \vec{D} perpendicular to \vec{A} is $D_\perp = D \sin 53.0^\circ = 7.99 \text{ m}$.
 $|\vec{A} \times \vec{D}| = AD_\perp = 63.9 \text{ m}^2$, which agrees with our previous result.
- 1.50. IDENTIFY:** The right-hand rule gives the direction and Eq. (1.22) gives the magnitude.
SET UP: $\phi = 120.0^\circ$.
EXECUTE: (a) The direction of $\vec{A} \times \vec{B}$ is into the page (the $-z$ -direction). The magnitude of the vector product is $AB \sin \phi = (2.80 \text{ cm})(1.90 \text{ cm}) \sin 120^\circ = 4.61 \text{ cm}^2$.
(b) Rather than repeat the calculations, Eq. (1.23) may be used to see that $\vec{B} \times \vec{A}$ has magnitude 4.61 cm^2 and is in the $+z$ -direction (out of the page).
EVALUATE: For part (a) we could use Eq. (1.27) and note that the only non-vanishing component is $C_z = A_x B_y - A_y B_x = (2.80 \text{ cm}) \cos 60.0^\circ (-1.90 \text{ cm}) \sin 60^\circ - (2.80 \text{ cm}) \sin 60.0^\circ (1.90 \text{ cm}) \cos 60.0^\circ = -4.61 \text{ cm}^2$.
 This gives the same result.
- 1.51. IDENTIFY:** Apply Eqs. (1.18) and (1.22).
SET UP: The angle between the vectors is $20^\circ + 90^\circ + 30^\circ = 140^\circ$.
EXECUTE: (a) Eq. (1.18) gives $\vec{A} \cdot \vec{B} = (3.60 \text{ m})(2.40 \text{ m}) \cos 140^\circ = -6.62 \text{ m}^2$.
(b) From Eq. (1.22), the magnitude of the cross product is $(3.60 \text{ m})(2.40 \text{ m}) \sin 140^\circ = 5.55 \text{ m}^2$ and the direction, from the right-hand rule, is out of the page (the $+z$ -direction).
EVALUATE: We could also use Eqs. (1.21) and (1.27), with the components of \vec{A} and \vec{B} .
- 1.52. IDENTIFY:** Use Eq. (1.27) for the components of the vector product.
SET UP: Use coordinates with the $+x$ -axis to the right, $+y$ -axis toward the top of the page, and $+z$ -axis out of the page. $A_x = 0$, $A_y = 0$ and $A_z = -3.50 \text{ cm}$. The page is 20 cm by 35 cm, so $B_x = -20 \text{ cm}$ and $B_y = 35 \text{ cm}$.
EXECUTE: $(\vec{A} \times \vec{B})_x = 122 \text{ cm}^2$, $(\vec{A} \times \vec{B})_y = 70 \text{ cm}^2$, $(\vec{A} \times \vec{B})_z = 0$.
EVALUATE: From the components we calculated the magnitude of the vector product is 141 cm^2 .
 $B = 40.3 \text{ cm}$ and $\phi = 90^\circ$, so $AB \sin \phi = 141 \text{ cm}^2$, which agrees.

1.53. IDENTIFY: \vec{A} and \vec{B} are given in unit vector form. Find A , B and the vector difference $\vec{A} - \vec{B}$.

SET UP: $\vec{A} = -2.00\vec{i} + 3.00\vec{j} + 4.00\vec{k}$, $\vec{B} = 3.00\vec{i} + 1.00\vec{j} - 3.00\vec{k}$

Use Eq. (1.8) to find the magnitudes of the vectors.

EXECUTE: (a) $A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{(-2.00)^2 + (3.00)^2 + (4.00)^2} = 5.38$

$$B = \sqrt{B_x^2 + B_y^2 + B_z^2} = \sqrt{(3.00)^2 + (1.00)^2 + (-3.00)^2} = 4.36$$

(b) $\vec{A} - \vec{B} = (-2.00\hat{i} + 3.00\hat{j} + 4.00\hat{k}) - (3.00\hat{i} + 1.00\hat{j} - 3.00\hat{k})$

$$\vec{A} - \vec{B} = (-2.00 - 3.00)\hat{i} + (3.00 - 1.00)\hat{j} + (4.00 - (-3.00))\hat{k} = -5.00\hat{i} + 2.00\hat{j} + 7.00\hat{k}$$

(c) Let $\vec{C} = \vec{A} - \vec{B}$, so $C_x = -5.00$, $C_y = +2.00$, $C_z = +7.00$

$$C = \sqrt{C_x^2 + C_y^2 + C_z^2} = \sqrt{(-5.00)^2 + (2.00)^2 + (7.00)^2} = 8.83$$

$\vec{B} - \vec{A} = -(\vec{A} - \vec{B})$, so $\vec{A} - \vec{B}$ and $\vec{B} - \vec{A}$ have the same magnitude but opposite directions.

EVALUATE: A , B and C are each larger than any of their components.

1.54. IDENTIFY: Area is length times width. Do unit conversions.

SET UP: 1 mi = 5280 ft. 1 ft³ = 7.477 gal.

EXECUTE: (a) The area of one acre is $\frac{1}{8} \text{ mi} \times \frac{1}{80} \text{ mi} = \frac{1}{640} \text{ mi}^2$, so there are 640 acres to a square mile.

$$(b) (1 \text{ acre}) \times \left(\frac{1 \text{ mi}^2}{640 \text{ acre}} \right) \times \left(\frac{5280 \text{ ft}}{1 \text{ mi}} \right)^2 = 43,560 \text{ ft}^2$$

(all of the above conversions are exact).

(c) (1 acre-foot) = $(43,560 \text{ ft}^3) \times \left(\frac{7.477 \text{ gal}}{1 \text{ ft}^3} \right) = 3.26 \times 10^5 \text{ gal}$, which is rounded to three significant figures.

EVALUATE: An acre is much larger than a square foot but less than a square mile. A volume of 1 acre-foot is much larger than a gallon.

1.55. IDENTIFY: The density relates mass and volume. Use the given mass and density to find the volume and from this the radius.

SET UP: The earth has mass $m_E = 5.97 \times 10^{24} \text{ kg}$ and radius $r_E = 6.38 \times 10^6 \text{ m}$. The volume of a sphere is $V = \frac{4}{3}\pi r^3$. $\rho = 1.76 \text{ g/cm}^3 = 1760 \text{ kg/m}^3$.

EXECUTE: (a) The planet has mass $m = 5.5m_E = 3.28 \times 10^{25} \text{ kg}$. $V = \frac{m}{\rho} = \frac{3.28 \times 10^{25} \text{ kg}}{1760 \text{ kg/m}^3} = 1.86 \times 10^{22} \text{ m}^3$.

$$r = \left(\frac{3V}{4\pi} \right)^{1/3} = \left(\frac{3[1.86 \times 10^{22} \text{ m}^3]}{4\pi} \right)^{1/3} = 1.64 \times 10^7 \text{ m} = 1.64 \times 10^4 \text{ km}$$

(b) $r = 2.57r_E$

EVALUATE: Volume V is proportional to mass and radius r is proportional to $V^{1/3}$, so r is proportional to $m^{1/3}$. If the planet and earth had the same density its radius would be $(5.5)^{1/3}r_E = 1.8r_E$. The radius of the planet is greater than this, so its density must be less than that of the earth.

1.56. IDENTIFY and SET UP: Unit conversion.

EXECUTE: (a) $f = 1.420 \times 10^9 \text{ cycles/s}$, so $\frac{1}{1.420 \times 10^9} \text{ s} = 7.04 \times 10^{-10} \text{ s}$ for one cycle.

$$(b) \frac{3600 \text{ s/h}}{7.04 \times 10^{-10} \text{ s/cycle}} = 5.11 \times 10^{12} \text{ cycles/h}$$

(c) Calculate the number of seconds in 4600 million years = $4.6 \times 10^9 \text{ y}$ and divide by the time for 1 cycle:

$$\frac{(4.6 \times 10^9 \text{ y})(3.156 \times 10^7 \text{ s/y})}{7.04 \times 10^{-10} \text{ s/cycle}} = 2.1 \times 10^{26} \text{ cycles}$$

(d) The clock is off by 1 s in $100,000 \text{ y} = 1 \times 10^5 \text{ y}$, so in $4.60 \times 10^9 \text{ y}$ it is off by

$$(1 \text{ s}) \left(\frac{4.60 \times 10^9}{1 \times 10^5} \right) = 4.6 \times 10^4 \text{ s (about 13 h)}.$$

EVALUATE: In each case the units in the calculation combine algebraically to give the correct units for the answer.

1.57. IDENTIFY: Using the density of the oxygen and volume of a breath, we want the mass of oxygen (the target variable in part (a)) breathed in per day and the dimensions of the tank in which it is stored.

SET UP: The mass is the density times the volume. Estimate 12 breaths per minute. We know $1 \text{ day} = 24 \text{ h}$, $1 \text{ h} = 60 \text{ min}$ and $1000 \text{ L} = 1 \text{ m}^3$. The volume of a cube having faces of length l is $V = l^3$.

EXECUTE: (a) $(12 \text{ breaths/min}) \left(\frac{60 \text{ min}}{1 \text{ h}} \right) \left(\frac{24 \text{ h}}{1 \text{ day}} \right) = 17,280 \text{ breaths/day}$. The volume of air breathed in

one day is $(\frac{1}{2} \text{ L/breath})(17,280 \text{ breaths/day}) = 8640 \text{ L} = 8.64 \text{ m}^3$. The mass of air breathed in one day is the density of air times the volume of air breathed: $m = (1.29 \text{ kg/m}^3)(8.64 \text{ m}^3) = 11.1 \text{ kg}$. As 20% of this quantity is oxygen, the mass of oxygen breathed in 1 day is $(0.20)(11.1 \text{ kg}) = 2.2 \text{ kg} = 2200 \text{ g}$.

(b) $V = 8.64 \text{ m}^3$ and $V = l^3$, so $l = V^{1/3} = 2.1 \text{ m}$.

EVALUATE: A person could not survive one day in a closed tank of this size because the exhaled air is breathed back into the tank and thus reduces the percent of oxygen in the air in the tank. That is, a person cannot extract all of the oxygen from the air in an enclosed space.

1.58. IDENTIFY: Use the extreme values in the piece's length and width to find the uncertainty in the area.

SET UP: The length could be as large as 7.61 cm and the width could be as large as 1.91 cm.

EXECUTE: The area is $14.44 \pm 0.095 \text{ cm}^2$. The fractional uncertainty in the area is $\frac{0.095 \text{ cm}^2}{14.44 \text{ cm}^2} = 0.66\%$,

and the fractional uncertainties in the length and width are $\frac{0.01 \text{ cm}}{7.61 \text{ cm}} = 0.13\%$ and $\frac{0.01 \text{ cm}}{1.9 \text{ cm}} = 0.53\%$. The sum of these fractional uncertainties is $0.13\% + 0.53\% = 0.66\%$, in agreement with the fractional uncertainty in the area.

EVALUATE: The fractional uncertainty in a product of numbers is greater than the fractional uncertainty in any of the individual numbers.

1.59. IDENTIFY: Calculate the average volume and diameter and the uncertainty in these quantities.

SET UP: Using the extreme values of the input data gives us the largest and smallest values of the target variables and from these we get the uncertainty.

EXECUTE: (a) The volume of a disk of diameter d and thickness t is $V = \pi(d/2)^2 t$.

The average volume is $V = \pi(8.50 \text{ cm}/2)^2(0.50 \text{ cm}) = 2.837 \text{ cm}^3$. But t is given to only two significant figures so the answer should be expressed to two significant figures: $V = 2.8 \text{ cm}^3$.

We can find the uncertainty in the volume as follows. The volume could be as large as

$V = \pi(8.52 \text{ cm}/2)^2(0.055 \text{ cm}) = 3.1 \text{ cm}^3$, which is 0.3 cm^3 larger than the average value. The volume

could be as small as $V = \pi(8.48 \text{ cm}/2)^2(0.045 \text{ cm}) = 2.5 \text{ cm}^3$, which is 0.3 cm^3 smaller than the average value. The uncertainty is $\pm 0.3 \text{ cm}^3$, and we express the volume as $V = 2.8 \pm 0.3 \text{ cm}^3$.

(b) The ratio of the average diameter to the average thickness is $8.50 \text{ cm}/0.050 \text{ cm} = 170$. By taking the largest possible value of the diameter and the smallest possible thickness we get the largest possible value for this ratio: $8.52 \text{ cm}/0.045 \text{ cm} = 190$. The smallest possible value of the ratio is $8.48/0.055 = 150$. Thus the uncertainty is ± 20 and we write the ratio as 170 ± 20 .

EVALUATE: The thickness is uncertain by 10% and the percentage uncertainty in the diameter is much less, so the percentage uncertainty in the volume and in the ratio should be about 10%.

- 1.60. IDENTIFY:** Estimate the volume of each object. The mass m is the density times the volume.
SET UP: The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$. The volume of a cylinder of radius r and length l is $V = \pi r^2 l$. The density of water is 1000 kg/m^3 .
EXECUTE: (a) Estimate the volume as that of a sphere of diameter 10 cm: $V = 5.2 \times 10^{-4} \text{ m}^3$.
 $m = (0.98)(1000 \text{ kg/m}^3)(5.2 \times 10^{-4} \text{ m}^3) = 0.5 \text{ kg}$.
 (b) Approximate as a sphere of radius $r = 0.25 \mu\text{m}$ (probably an overestimate): $V = 6.5 \times 10^{-20} \text{ m}^3$.
 $m = (0.98)(1000 \text{ kg/m}^3)(6.5 \times 10^{-20} \text{ m}^3) = 6 \times 10^{-17} \text{ kg} = 6 \times 10^{-14} \text{ g}$.
 (c) Estimate the volume as that of a cylinder of length 1 cm and radius 3 mm: $V = \pi r^2 l = 2.8 \times 10^{-7} \text{ m}^3$.
 $m = (0.98)(1000 \text{ kg/m}^3)(2.8 \times 10^{-7} \text{ m}^3) = 3 \times 10^{-4} \text{ kg} = 0.3 \text{ g}$.
EVALUATE: The mass is directly proportional to the volume.
- 1.61. IDENTIFY:** The number of atoms is your mass divided by the mass of one atom.
SET UP: Assume a 70-kg person and that the human body is mostly water. Use Appendix D to find the mass of one H_2O molecule: $18.015 \text{ u} \times 1.661 \times 10^{-27} \text{ kg/u} = 2.992 \times 10^{-26} \text{ kg/molecule}$.
EXECUTE: $(70 \text{ kg}) / (2.992 \times 10^{-26} \text{ kg/molecule}) = 2.34 \times 10^{27}$ molecules. Each H_2O molecule has 3 atoms, so there are about 6×10^{27} atoms.
EVALUATE: Assuming carbon to be the most common atom gives 3×10^{27} molecules, which is a result of the same order of magnitude.
- 1.62. IDENTIFY:** The number of bills is the distance to the moon divided by the thickness of one bill.
SET UP: Estimate the thickness of a dollar bill by measuring a short stack, say ten, and dividing the measurement by the total number of bills. I obtain a thickness of roughly 1 mm. From Appendix F, the distance from the earth to the moon is $3.8 \times 10^8 \text{ m}$.
EXECUTE: $N_{\text{bills}} = \left(\frac{3.8 \times 10^8 \text{ m}}{0.1 \text{ mm/bill}} \right) \left(\frac{10^3 \text{ mm}}{1 \text{ m}} \right) = 3.8 \times 10^{12} \text{ bills} \approx 4 \times 10^{12} \text{ bills}$
EVALUATE: This answer represents 4 trillion dollars! The cost of a single space shuttle mission in 2005 is significantly less—roughly 1 billion dollars.
- 1.63. IDENTIFY:** The cost would equal the number of dollar bills required; the surface area of the U.S. divided by the surface area of a single dollar bill.
SET UP: By drawing a rectangle on a map of the U.S., the approximate area is 2600 mi by 1300 mi or $3,380,000 \text{ mi}^2$. This estimate is within 10 percent of the actual area, $3,794,083 \text{ mi}^2$. The population is roughly 3.0×10^8 while the area of a dollar bill, as measured with a ruler, is approximately $6\frac{1}{8} \text{ in.}$ by $2\frac{5}{8} \text{ in.}$
EXECUTE: $A_{\text{U.S.}} = (3,380,000 \text{ mi}^2)[(5280 \text{ ft})/(1 \text{ mi})]^2[(12 \text{ in.})/(1 \text{ ft})]^2 = 1.4 \times 10^{16} \text{ in.}^2$
 $A_{\text{bill}} = (6.125 \text{ in.})(2.625 \text{ in.}) = 16.1 \text{ in.}^2$
 Total cost = $N_{\text{bills}} = A_{\text{U.S.}}/A_{\text{bill}} = (1.4 \times 10^{16} \text{ in.}^2)/(16.1 \text{ in.}^2/\text{bill}) = 9 \times 10^{14} \text{ bills}$
 Cost per person = $(9 \times 10^{14} \text{ dollars})/(3.0 \times 10^8 \text{ persons}) = 3 \times 10^6 \text{ dollars/person}$
EVALUATE: The actual cost would be somewhat larger, because the land isn't flat.
- 1.64. IDENTIFY:** Estimate the volume of sand in all the beaches on the earth. The diameter of a grain of sand determines its volume. From the volume of one grain and the total volume of sand we can calculate the number of grains.
SET UP: The volume of a sphere of diameter d is $V = \frac{1}{6}\pi d^3$. Consulting an atlas, we estimate that the continents have about $1.45 \times 10^5 \text{ km}$ of coastline. Add another 25% of this for rivers and lakes, giving $1.82 \times 10^5 \text{ km}$ of coastline. Assume that a beach extends 50 m beyond the water and that the sand is 2 m deep. 1 billion = 1×10^9 .

EXECUTE: (a) The volume of sand is $(1.82 \times 10^8 \text{ m})(50 \text{ m})(2 \text{ m}) = 2 \times 10^{10} \text{ m}^3$. The volume of a grain is $V = \frac{1}{6}\pi(0.2 \times 10^{-3} \text{ m})^3 = 4 \times 10^{-12} \text{ m}^3$. The number of grains is $\frac{2 \times 10^{10} \text{ m}^3}{4 \times 10^{-12} \text{ m}^3} = 5 \times 10^{21}$. The number of grains of sand is about 10^{22} .

(b) The number of stars is $(100 \times 10^9)(100 \times 10^9) = 10^{22}$. The two estimates result in comparable numbers for these two quantities.

EVALUATE: Both numbers are crude estimates but are probably accurate to a few powers of 10.

- 1.65. IDENTIFY:** We know the magnitude and direction of the sum of the two vector pulls and the direction of one pull. We also know that one pull has twice the magnitude of the other. There are two unknowns, the magnitude of the smaller pull and its direction. $A_x + B_x = C_x$ and $A_y + B_y = C_y$ give two equations for these two unknowns.

SET UP: Let the smaller pull be \vec{A} and the larger pull be \vec{B} . $B = 2A$. $\vec{C} = \vec{A} + \vec{B}$ has magnitude 460.0 N and is northward. Let $+x$ be east and $+y$ be north. $B_x = -B \sin 25.0^\circ$ and $B_y = B \cos 25.0^\circ$. $C_x = 0$, $C_y = 460.0 \text{ N}$. \vec{A} must have an eastward component to cancel the westward component of \vec{B} . There are then two possibilities, as sketched in Figures 1.65 a and b. \vec{A} can have a northward component or \vec{A} can have a southward component.

EXECUTE: In either Figure 1.65 a or b, $A_x + B_x = C_x$ and $B = 2A$ gives $(2A) \sin 25.0^\circ = A \sin \phi$ and $\phi = 57.7^\circ$. In Figure 1.65a, $A_y + B_y = C_y$ gives $2A \cos 25.0^\circ + A \cos 57.7^\circ = 460.0 \text{ N}$ and $A = 196 \text{ N}$. In Figure 1.65b, $2A \cos 25.0^\circ - A \cos 57.7^\circ = 460.0 \text{ N}$ and $A = 360 \text{ N}$. One solution is for the smaller pull to be 57.7° east of north. In this case, the smaller pull is 196 N and the larger pull is 392 N. The other solution is for the smaller pull to be 57.7° east of south. In this case the smaller pull is 360 N and the larger pull is 720 N.

EVALUATE: For the first solution, with \vec{A} east of north, each worker has to exert less force to produce the given resultant force and this is the sensible direction for the worker to pull.

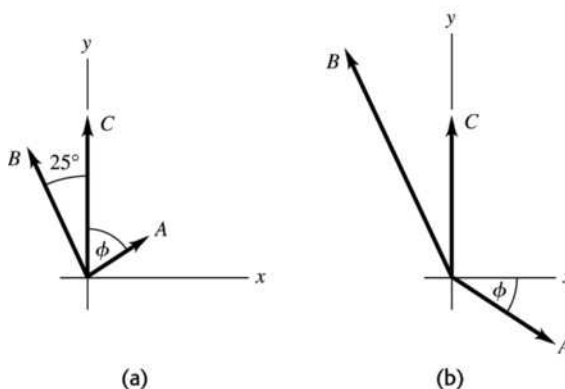


Figure 1.65

- 1.66. IDENTIFY:** Let \vec{D} be the fourth force. Find \vec{D} such that $\vec{A} + \vec{B} + \vec{C} + \vec{D} = 0$, so $\vec{D} = -(\vec{A} + \vec{B} + \vec{C})$.

SET UP: Use components and solve for the components D_x and D_y of \vec{D} .

EXECUTE: $A_x = +A \cos 30.0^\circ = +86.6 \text{ N}$, $A_y = +A \sin 30.0^\circ = +50.00 \text{ N}$.

$B_x = -B \sin 30.0^\circ = -40.00 \text{ N}$, $B_y = +B \cos 30.0^\circ = +69.28 \text{ N}$.

$C_x = -C \cos 53.0^\circ = -24.07 \text{ N}$, $C_y = -C \sin 53.0^\circ = -31.90 \text{ N}$.

Then $D_x = -22.53 \text{ N}$, $D_y = -87.34 \text{ N}$ and $D = \sqrt{D_x^2 + D_y^2} = 90.2 \text{ N}$. $\tan \alpha = |D_y/D_x| = 87.34/22.53$.
 $\alpha = 75.54^\circ$. $\phi = 180^\circ + \alpha = 256^\circ$, counterclockwise from the $+x$ -axis.

EVALUATE: As shown in Figure 1.66, since D_x and D_y are both negative, \vec{D} must lie in the third quadrant.

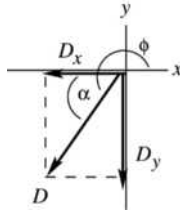
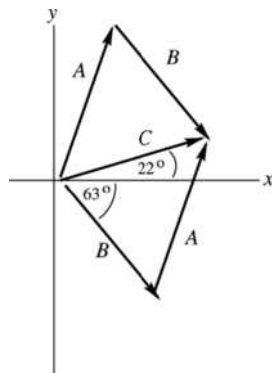


Figure 1.66

1.67. **IDENTIFY:** $\vec{A} + \vec{B} = \vec{C}$ (or $\vec{B} + \vec{A} = \vec{C}$). The target variable is vector \vec{A} .

SET UP: Use components and Eq. (1.10) to solve for the components of \vec{A} . Find the magnitude and direction of \vec{A} from its components.

EXECUTE: (a)



$$C_x = A_x + B_x, \text{ so } A_x = C_x - B_x$$

$$C_y = A_y + B_y, \text{ so } A_y = C_y - B_y$$

$$C_x = C \cos 22.0^\circ = (6.40 \text{ cm}) \cos 22.0^\circ$$

$$C_x = +5.934 \text{ cm}$$

$$C_y = C \sin 22.0^\circ = (6.40 \text{ cm}) \sin 22.0^\circ$$

$$C_y = +2.397 \text{ cm}$$

$$B_x = B \cos (360^\circ - 63.0^\circ) = (6.40 \text{ cm}) \cos 297.0^\circ$$

$$B_x = +2.906 \text{ cm}$$

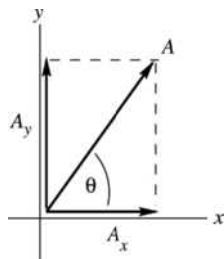
$$B_y = B \sin 297.0^\circ = (6.40 \text{ cm}) \sin 297.0^\circ$$

$$B_y = -5.702 \text{ cm}$$

Figure 1.67a

(b) $A_x = C_x - B_x = +5.934 \text{ cm} - 2.906 \text{ cm} = +3.03 \text{ cm}$

$A_y = C_y - B_y = +2.397 \text{ cm} - (-5.702) \text{ cm} = +8.10 \text{ cm}$



$$A = \sqrt{A_x^2 + A_y^2}$$

$$A = \sqrt{(3.03 \text{ cm})^2 + (8.10 \text{ cm})^2} = 8.65 \text{ cm}$$

$$\tan \theta = \frac{A_y}{A_x} = \frac{8.10 \text{ cm}}{3.03 \text{ cm}} = 2.67$$

$$\theta = 69.5^\circ$$

Figure 1.67b

EVALUATE: The \vec{A} we calculated agrees qualitatively with vector \vec{A} in the vector addition diagram in part (a).

1.68. **IDENTIFY:** Find the vector sum of the two displacements.

SET UP: Call the two displacements \vec{A} and \vec{B} , where $A = 170 \text{ km}$ and $B = 230 \text{ km}$. $\vec{A} + \vec{B} = \vec{R}$. \vec{A} and \vec{B} are as shown in Figure 1.68.

EXECUTE: $R_x = A_x + B_x = (170 \text{ km})\sin 68^\circ + (230 \text{ km})\cos 48^\circ = 311.5 \text{ km}$.

$R_y = A_y + B_y = (170 \text{ km})\cos 68^\circ - (230 \text{ km})\sin 48^\circ = -107.2 \text{ km}$.

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(311.5 \text{ km})^2 + (-107.2 \text{ km})^2} = 330 \text{ km}$. $\tan \theta_R = \left| \frac{R_y}{R_x} \right| = \frac{107.2 \text{ km}}{311.5 \text{ km}} = 0.344$.

$\theta_R = 19^\circ$ south of east.

EVALUATE: Our calculation using components agrees with \vec{R} shown in the vector addition diagram, Figure 1.68.

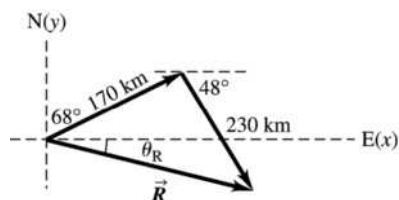


Figure 1.68

1.69. IDENTIFY: Vector addition. Target variable is the 4th displacement.

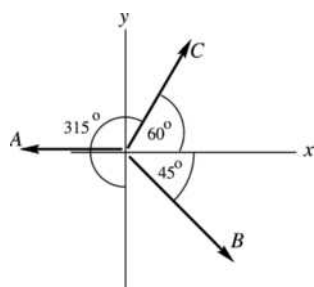
SET UP: Use a coordinate system where east is in the $+x$ -direction and north is in the $+y$ -direction.

Let \vec{A} , \vec{B} , and \vec{C} be the three displacements that are given and let \vec{D} be the fourth unmeasured displacement. Then the resultant displacement is $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$. And since she ends up back where she started, $\vec{R} = 0$.

$0 = \vec{A} + \vec{B} + \vec{C} + \vec{D}$, so $\vec{D} = -(\vec{A} + \vec{B} + \vec{C})$

$D_x = -(A_x + B_x + C_x)$ and $D_y = -(A_y + B_y + C_y)$

EXECUTE:



$A_x = -180 \text{ m}$, $A_y = 0$

$B_x = B \cos 315^\circ = (210 \text{ m})\cos 315^\circ = +148.5 \text{ m}$

$B_y = B \sin 315^\circ = (210 \text{ m})\sin 315^\circ = -148.5 \text{ m}$

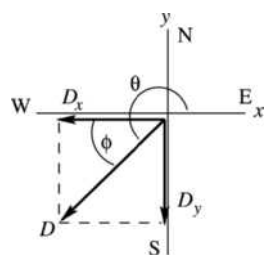
$C_x = C \cos 60^\circ = (280 \text{ m})\cos 60^\circ = +140 \text{ m}$

$C_y = C \sin 60^\circ = (280 \text{ m})\sin 60^\circ = +242.5 \text{ m}$

Figure 1.69a

$D_x = -(A_x + B_x + C_x) = -(-180 \text{ m} + 148.5 \text{ m} + 140 \text{ m}) = -108.5 \text{ m}$

$D_y = -(A_y + B_y + C_y) = -(0 - 148.5 \text{ m} + 242.5 \text{ m}) = -94.0 \text{ m}$



$D = \sqrt{D_x^2 + D_y^2}$

$D = \sqrt{(-108.5 \text{ m})^2 + (-94.0 \text{ m})^2} = 144 \text{ m}$

$\tan \theta = \frac{D_y}{D_x} = \frac{-94.0 \text{ m}}{-108.5 \text{ m}} = 0.8664$

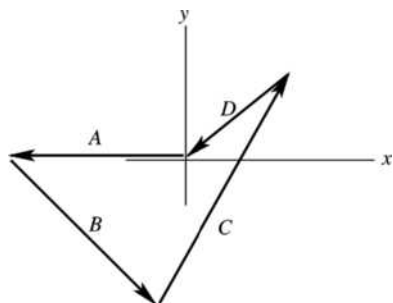
$\theta = 180^\circ + 40.9^\circ = 220.9^\circ$

(\vec{D} is in the third quadrant since both D_x and D_y are negative.)

Figure 1.69b

The direction of \vec{D} can also be specified in terms of $\phi = \theta - 180^\circ = 40.9^\circ$; \vec{D} is 41° south of west.

EVALUATE: The vector addition diagram, approximately to scale, is



Vector \vec{D} in this diagram agrees qualitatively with our calculation using components.

Figure 1.69c

1.70. IDENTIFY: Add the vectors using the method of components.

SET UP: $A_x = 0$, $A_y = -8.00$ m. $B_x = 7.50$ m, $B_y = 13.0$ m. $C_x = -10.9$ m, $C_y = -5.07$ m.

EXECUTE: (a) $R_x = A_x + B_x + C_x = -3.4$ m. $R_y = A_y + B_y + C_y = -0.07$ m. $R = 3.4$ m. $\tan \theta = \frac{-0.07 \text{ m}}{-3.4 \text{ m}}$.
 $\theta = 1.2^\circ$ below the $-x$ -axis.

(b) $S_x = C_x - A_x - B_x = -18.4$ m. $S_y = C_y - A_y - B_y = -10.1$ m. $S = 21.0$ m. $\tan \theta = \frac{S_y}{S_x} = \frac{-10.1 \text{ m}}{-18.4 \text{ m}}$.

$\theta = 28.8^\circ$ below the $-x$ -axis.

EVALUATE: The magnitude and direction we calculated for \vec{R} and \vec{S} agree with our vector diagrams.

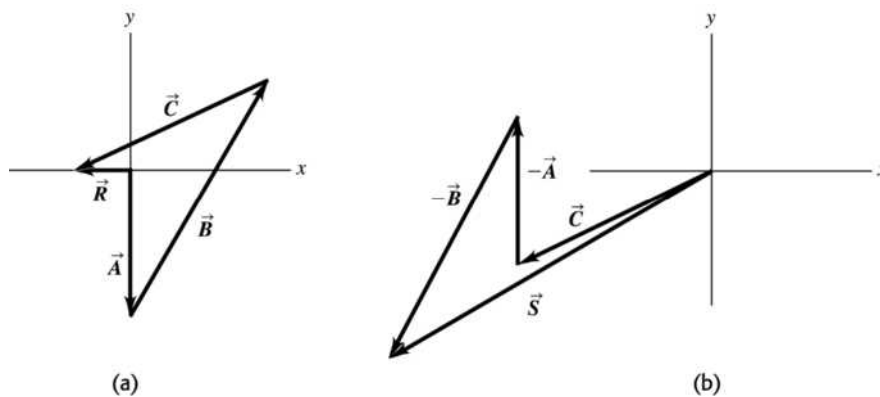


Figure 1.70

1.71. IDENTIFY: Find the vector sum of the two forces.

SET UP: Use components to add the two forces. Take the $+x$ -direction to be forward and the $+y$ -direction to be upward.

EXECUTE: The second force has components $F_{2x} = F_2 \cos 32.4^\circ = 433$ N and $F_{2y} = F_2 \sin 32.4^\circ = 275$ N.

The first force has components $F_{1x} = 480$ N and $F_{1y} = 0$. $F_x = F_{1x} + F_{2x} = 913$ N and

$F_y = F_{1y} + F_{2y} = 275$ N. The resultant force is 954 N in the direction 16.8° above the forward direction.

EVALUATE: Since the two forces are not in the same direction the magnitude of their vector sum is less than the sum of their magnitudes.

1.72. IDENTIFY: Solve for one of the vectors in the vector sum. Use components.

SET UP: Use coordinates for which $+x$ is east and $+y$ is north. The vector displacements are:

$\vec{A} = 2.00$ km, 0° of east; $\vec{B} = 3.50$ m, 45° south of east; and $\vec{R} = 5.80$ m, 0° east

EXECUTE: $C_x = R_x - A_x - B_x = 5.80 \text{ km} - (2.00 \text{ km}) - (3.50 \text{ km})(\cos 45^\circ) = 1.33 \text{ km}$; $C_y = R_y - A_y - B_y = 0 \text{ km} - 0 \text{ km} - (-3.50 \text{ km})(\sin 45^\circ) = 2.47 \text{ km}$; $C = \sqrt{(1.33 \text{ km})^2 + (2.47 \text{ km})^2} = 2.81 \text{ km}$;

$\theta = \tan^{-1}[(2.47 \text{ km})/(1.33 \text{ km})] = 61.7^\circ$ north of east. The vector addition diagram in Figure 1.72 shows good qualitative agreement with these values.

EVALUATE: The third leg lies in the first quadrant since its x and y components are both positive.

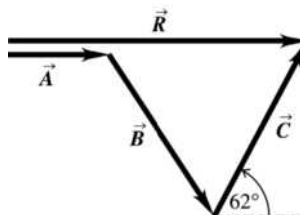


Figure 1.72

- 1.73. IDENTIFY:** We know the resultant of two forces of known equal magnitudes and want to find that magnitude (the target variable).

SET UP: Use coordinates having a horizontal $+x$ axis and an upward $+y$ axis. Then $A_x + B_x = R_x$ and $R_x = 5.60 \text{ N}$.

SOLVE: $A_x + B_x = R_x$ and $A \cos 32^\circ + B \sin 32^\circ = R_x$. Since $A = B$,

$$2A \cos 32^\circ = R_x, \text{ so } A = \frac{R_x}{2(\cos 32^\circ)} = 3.30 \text{ N}.$$

EVALUATE: The magnitude of the x component of each pull is 2.80 N , so the magnitude of each pull (3.30 N) is greater than its x component, as it should be.

- 1.74. IDENTIFY:** The four displacements return her to her starting point, so $\vec{D} = -(\vec{A} + \vec{B} + \vec{C})$, where \vec{A} , \vec{B} and \vec{C} are in the three given displacements and \vec{D} is the displacement for her return.

START UP: Let $+x$ be east and $+y$ be north.

EXECUTE: (a) $D_x = -[(147 \text{ km})\sin 85^\circ + (106 \text{ km})\sin 167^\circ + (166 \text{ km})\sin 235^\circ] = -34.3 \text{ km}$.

$$D_y = -[(147 \text{ km})\cos 85^\circ + (106 \text{ km})\cos 167^\circ + (166 \text{ km})\cos 235^\circ] = +185.7 \text{ km}.$$

$$D = \sqrt{(-34.3 \text{ km})^2 + (185.7 \text{ km})^2} = 189 \text{ km}.$$

(b) The direction relative to north is $\phi = \arctan\left(\frac{34.3 \text{ km}}{185.7 \text{ km}}\right) = 10.5^\circ$. Since $D_x < 0$ and $D_y > 0$, the

direction of \vec{D} is 10.5° west of north.

EVALUATE: The four displacements add to zero.

- 1.75. IDENTIFY:** The sum of the vector forces on the beam sum to zero, so their x components and their y components sum to zero. Solve for the components of \vec{F} .

SET UP: The forces on the beam are sketched in Figure 1.75a. Choose coordinates as shown in the sketch. The 100-N pull makes an angle of $30.0^\circ + 40.0^\circ = 70.0^\circ$ with the horizontal. \vec{F} and the 100-N pull have been replaced by their x and y components.

EXECUTE: (a) The sum of the x -components is equal to zero gives $F_x + (100 \text{ N})\cos 70.0^\circ = 0$ and

$$F_x = -34.2 \text{ N}.$$

The sum of the y -components is equal to zero gives $F_y + (100 \text{ N})\sin 70.0^\circ - 124 \text{ N} = 0$ and

$$F_y = +30.0 \text{ N}.$$

\vec{F} and its components are sketched in Figure 1.75b. $F = \sqrt{F_x^2 + F_y^2} = 45.5 \text{ N}$.

$\tan \phi = \frac{|F_y|}{|F_x|} = \frac{30.0 \text{ N}}{34.2 \text{ N}}$ and $\phi = 41.3^\circ$. \vec{F} is directed at 41.3° above the $-x$ -axis in Figure 1.75a.

(b) The vector addition diagram is given in Figure 1.75c. \vec{F} determined from the diagram agrees with \vec{F} calculated in part (a) using components.

EVALUATE: The vertical component of the 100 N pull is less than the 124 N weight so \vec{F} must have an upward component if all three forces balance.

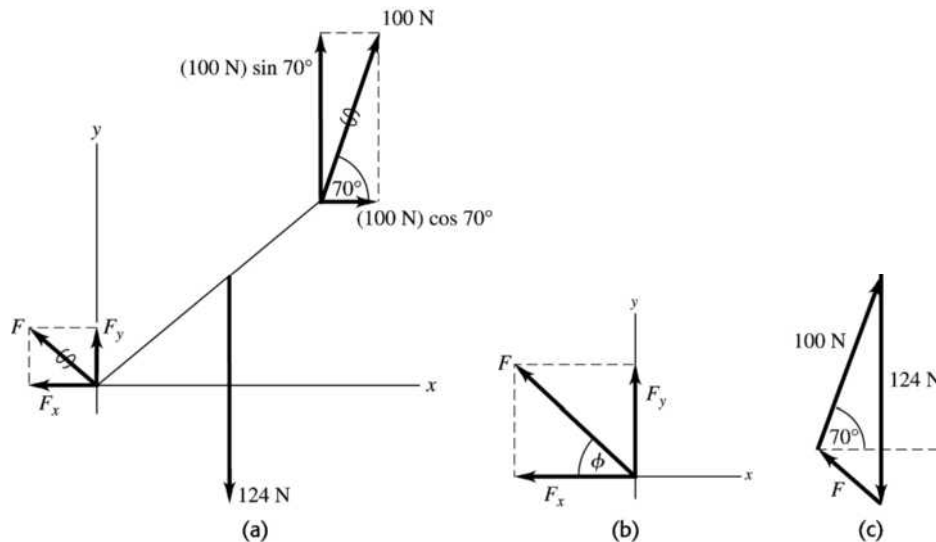


Figure 1.75

- 1.76. **IDENTIFY:** Let the three given displacements be \vec{A} , \vec{B} and \vec{C} , where $A = 40$ steps, $B = 80$ steps and $C = 50$ steps. $\vec{R} = \vec{A} + \vec{B} + \vec{C}$. The displacement \vec{C} that will return him to his hut is $-\vec{R}$.

SET UP: Let the east direction be the $+x$ -direction and the north direction be the $+y$ -direction.

EXECUTE: (a) The three displacements and their resultant are sketched in Figure 1.76.

(b) $R_x = (40)\cos 45^\circ - (80)\cos 60^\circ = -11.7$ and $R_y = (40)\sin 45^\circ + (80)\sin 60^\circ - 50 = 47.6$.

The magnitude and direction of the resultant are $\sqrt{(-11.7)^2 + (47.6)^2} = 49$, $\arctan\left(\frac{47.6}{-11.7}\right) = 76^\circ$, north of

west. We know that \vec{R} is in the second quadrant because $R_x < 0$, $R_y > 0$. To return to the hut, the explorer must take 49 steps in a direction 76° south of east, which is 14° east of south.

EVALUATE: It is useful to show R_x , R_y and \vec{R} on a sketch, so we can specify what angle we are computing.

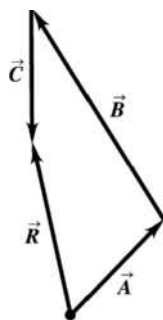


Figure 1.76

- 1.77. **IDENTIFY and SET UP:** The vector \vec{A} that connects points (x_1, y_1) and (x_2, y_2) has components $A_x = x_2 - x_1$ and $A_y = y_2 - y_1$.

EXECUTE: (a) Angle of first line is $\theta = \tan^{-1}\left(\frac{200-20}{210-10}\right) = 42^\circ$. Angle of second line is $42^\circ + 30^\circ = 72^\circ$.

Therefore $X = 10 + 250 \cos 72^\circ = 87$, $Y = 20 + 250 \sin 72^\circ = 258$ for a final point of $(87, 258)$.

(b) The computer screen now looks something like Figure 1.77. The length of the bottom line is $\sqrt{(210-87)^2 + (200-258)^2} = 136$ and its direction is $\tan^{-1}\left(\frac{258-200}{210-87}\right) = 25^\circ$ below straight left.

EVALUATE: Figure 1.77 is a vector addition diagram. The vector first line plus the vector arrow gives the vector for the second line.

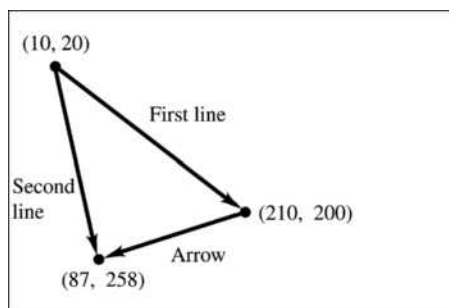


Figure 1.77

- 1.78. **IDENTIFY:** Vector addition. One vector and the sum are given; find the second vector (magnitude and direction).

SET UP: Let $+x$ be east and $+y$ be north. Let \vec{A} be the displacement 285 km at 40.0° north of west and let \vec{B} be the unknown displacement.

$$\vec{A} + \vec{B} = \vec{R} \text{ where } \vec{R} = 115 \text{ km, east}$$

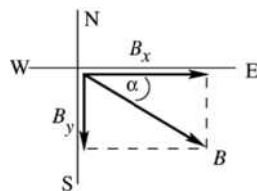
$$\vec{B} = \vec{R} - \vec{A}$$

$$B_x = R_x - A_x, \quad B_y = R_y - A_y$$

EXECUTE: $A_x = -A \cos 40.0^\circ = -218.3 \text{ km}$, $A_y = +A \sin 40.0^\circ = +183.2 \text{ km}$

$$R_x = 115 \text{ km}, \quad R_y = 0$$

Then $B_x = 333.3 \text{ km}$, $B_y = -183.2 \text{ km}$. $B = \sqrt{B_x^2 + B_y^2} = 380 \text{ km}$;



$$\tan \alpha = |B_y/B_x| = (183.2 \text{ km})/(333.3 \text{ km})$$

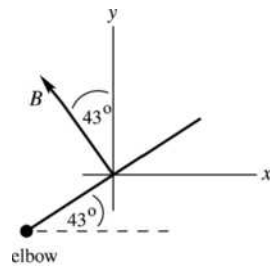
$$\alpha = 28.8^\circ, \text{ south of east}$$

Figure 1.78

EVALUATE: The southward component of \vec{B} cancels the northward component of \vec{A} . The eastward component of \vec{B} must be 115 km larger than the magnitude of the westward component of \vec{A} .

- 1.79. **IDENTIFY:** Vector addition. One force and the vector sum are given; find the second force.

SET UP: Use components. Let $+y$ be upward.



\vec{B} is the force the biceps exerts.

Figure 1.79a

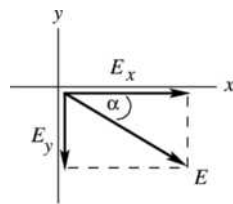
\vec{E} is the force the elbow exerts. $\vec{E} + \vec{B} = \vec{R}$, where $R = 132.5$ N and is upward.

$$E_x = R_x - B_x, E_y = R_y - B_y$$

EXECUTE: $B_x = -B \sin 43^\circ = -158.2$ N, $B_y = +B \cos 43^\circ = +169.7$ N, $R_x = 0$, $R_y = +132.5$ N

Then $E_x = +158.2$ N, $E_y = -37.2$ N.

$$E = \sqrt{E_x^2 + E_y^2} = 160$$
 N;



$$\tan \alpha = |E_y/E_x| = 37.2/158.2$$

$\alpha = 13^\circ$, below horizontal

Figure 1.79b

EVALUATE: The x -component of \vec{E} cancels the x -component of \vec{B} . The resultant upward force is less than the upward component of \vec{B} , so E_y must be downward.

1.80. IDENTIFY: Find the vector sum of the four displacements.

SET UP: Take the beginning of the journey as the origin, with north being the y -direction, east the x -direction, and the z -axis vertical. The first displacement is then $(-30 \text{ m})\hat{k}$, the second is $(-15 \text{ m})\hat{j}$, the third is $(200 \text{ m})\hat{i}$, and the fourth is $(100 \text{ m})\hat{j}$.

EXECUTE: (a) Adding the four displacements gives

$$(-30 \text{ m})\hat{k} + (-15 \text{ m})\hat{j} + (200 \text{ m})\hat{i} + (100 \text{ m})\hat{j} = (200 \text{ m})\hat{i} + (85 \text{ m})\hat{j} - (30 \text{ m})\hat{k}.$$

(b) The total distance traveled is the sum of the distances of the individual segments:

$$30 \text{ m} + 15 \text{ m} + 200 \text{ m} + 100 \text{ m} = 345 \text{ m}.$$

$$\text{The magnitude of the total displacement is: } D = \sqrt{D_x^2 + D_y^2 + D_z^2} = \sqrt{(200 \text{ m})^2 + (85 \text{ m})^2 + (-30 \text{ m})^2} = 219 \text{ m}.$$

EVALUATE: The magnitude of the displacement is much less than the distance traveled along the path.

1.81. IDENTIFY: The sum of the force displacements must be zero. Use components.

SET UP: Call the displacements \vec{A} , \vec{B} , \vec{C} and \vec{D} , where \vec{D} is the final unknown displacement for the return from the treasure to the oak tree. Vectors \vec{A} , \vec{B} , and \vec{C} are sketched in Figure 1.81a.

$\vec{A} + \vec{B} + \vec{C} + \vec{D} = 0$ says $A_x + B_x + C_x + D_x = 0$ and $A_y + B_y + C_y + D_y = 0$. $A = 825$ m, $B = 1250$ m, and $C = 1000$ m. Let $+x$ be eastward and $+y$ be north.

EXECUTE: (a) $A_x + B_x + C_x + D_x = 0$ gives

$$D_x = -(A_x + B_x + C_x) = -(0 - [1250 \text{ m}]\sin 30.0^\circ + [1000 \text{ m}]\cos 40.0^\circ) = -141 \text{ m}.$$

$A_y + B_y + C_y + D_y = 0$ gives $D_y = -(A_y + B_y + C_y) = -(-825 \text{ m} + [1250 \text{ m}]\cos 30.0^\circ + [1000 \text{ m}]\sin 40.0^\circ) = -900 \text{ m}$. The fourth

displacement \vec{D} and its components are sketched in Figure 1.81b. $D = \sqrt{D_x^2 + D_y^2} = 911 \text{ m}$.

$\tan \phi = \frac{|D_x|}{|D_y|} = \frac{141 \text{ m}}{900 \text{ m}}$ and $\phi = 8.9^\circ$. You should head 8.9° west of south and must walk 911 m.

(b) The vector diagram is sketched in Figure 1.81c. The final displacement \vec{D} from this diagram agrees with the vector \vec{D} calculated in part (a) using components.

EVALUATE: Note that \vec{D} is the negative of the sum of \vec{A} , \vec{B} , and \vec{C} .

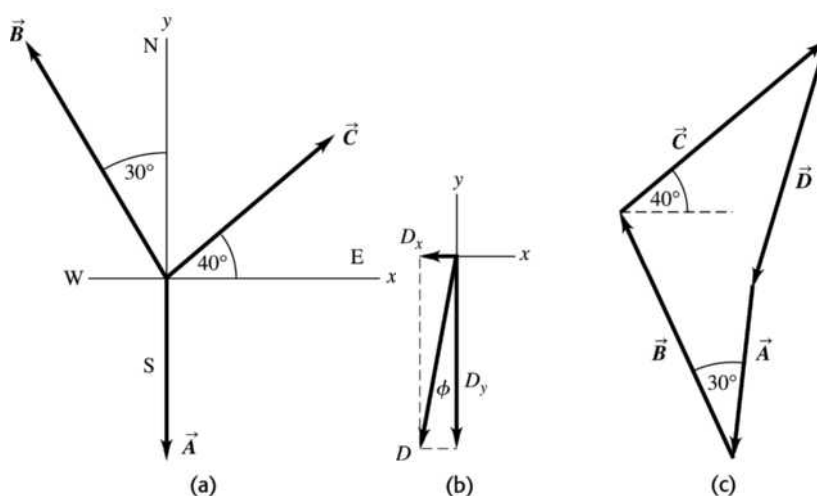


Figure 1.81

- 1.82. IDENTIFY:** The displacements are vectors in which we know the magnitude of the resultant and want to find the magnitude of one of the other vectors.

SET UP: Calling \vec{A} the vector from you to the first post, \vec{B} the vector from you to the second post, and \vec{C} the vector from the first to the second post, we have $\vec{A} + \vec{C} + \vec{B}$. Solving using components and the magnitude of \vec{C} gives $A_x + C_x = B_x$ and $A_y + C_y = B_y$.

EXECUTE: $B_x = 0$, $A_x = 41.53 \text{ m}$ and $C_x = B_x - A_x = -41.53 \text{ m}$.

$C = 80.0 \text{ m}$, so $C_y = \pm\sqrt{C^2 - C_x^2} = \pm 68.38 \text{ m}$.

The post is 37.1 m from you.

EVALUATE: $B_y = -37.1 \text{ m}$ (negative) since post is south of you (in the negative y direction).

- 1.83. IDENTIFY:** We are given the resultant of three vectors, two of which we know, and want to find the magnitude and direction of the third vector.

SET UP: Calling \vec{C} the unknown vector and \vec{A} and \vec{B} the known vectors, we have $\vec{A} + \vec{B} + \vec{C} = \vec{R}$. The components are $A_x + B_x + C_x = R_x$ and $A_y + B_y + C_y = R_y$.

EXECUTE: The components of the known vectors are $A_x = 12.0 \text{ m}$, $A_y = 0$,

$B_x = -B \sin 50.0^\circ = -21.45 \text{ m}$, $B_y = B \cos 50.0^\circ = +18.00 \text{ m}$, $R_x = 0$, and $R_y = -10.0 \text{ m}$. Therefore the

components of \vec{C} are $C_x = R_x - A_x - B_x = 0 - 12.0 \text{ m} - (-21.45 \text{ m}) = 9.45 \text{ m}$ and

$C_y = R_y - A_y - B_y = -10.0 \text{ m} - 0 - 18.0 \text{ m} = -28.0 \text{ m}$.

Using these components to find the magnitude and direction of \vec{C} gives $C = 29.6 \text{ m}$ and $\tan \theta = \frac{9.45}{28.0}$ and

$\theta = 18.6^\circ$ east of south

EVALUATE: A graphical sketch shows that this answer is reasonable.

- 1.84. IDENTIFY:** The displacements are vectors in which we know the magnitude of the resultant and want to find the magnitude of one of the other vectors.

SET UP: Calling \vec{A} the vector of Ricardo's displacement from the tree, \vec{B} the vector of Jane's displacement from the tree, and \vec{C} the vector from Ricardo to Jane, we have $\vec{A} + \vec{C} = \vec{B}$. Solving using components we have $A_x + C_x = B_x$ and $A_y + C_y = B_y$.

EXECUTE: (a) The components of \vec{A} and \vec{B} are $A_x = -(26.0 \text{ m})\sin 60.0^\circ = -22.52 \text{ m}$,
 $A_y = (26.0 \text{ m})\cos 60.0^\circ = +13.0 \text{ m}$, $B_x = -(16.0 \text{ m})\cos 30.0^\circ = -13.86 \text{ m}$,
 $B_y = -(16.0 \text{ m})\sin 30.0^\circ = -8.00 \text{ m}$, $C_x = B_x - A_x = -13.86 \text{ m} - (-22.52 \text{ m}) = +8.66 \text{ m}$,
 $C_y = B_y - A_y = -8.00 \text{ m} - (13.0 \text{ m}) = -21.0 \text{ m}$

Finding the magnitude from the components gives $C = 22.7 \text{ m}$.

(b) Finding the direction from the components gives $\tan \theta = \frac{8.66}{21.0}$ and $\theta = 22.4^\circ$, east of south.

EVALUATE: A graphical sketch confirms that this answer is reasonable.

- 1.85. IDENTIFY:** Think of the displacements of the three people as vectors. We know two of them and want to find their resultant.

SET UP: Calling \vec{A} the vector from John to Paul, \vec{B} the vector from Paul to George, and \vec{C} the vector from John to George, we have $\vec{A} + \vec{B} = \vec{C}$, which gives $A_x + B_x = C_x$ and $A_y + B_y = C_y$.

EXECUTE: The known components are $A_x = -14.0 \text{ m}$, $A_y = 0$, $B_x = B \cos 37^\circ = 28.75 \text{ m}$, and
 $B_y = -B \sin 37^\circ = -21.67 \text{ m}$. Therefore $C_x = -14.0 \text{ m} + 28.75 \text{ m} = 14.75 \text{ m}$, $C_y = 0 - 21.67 \text{ m} = -21.67 \text{ m}$.

These components give $C = 26.2 \text{ m}$ and $\tan \theta = \frac{14.75}{21.67}$, which gives $\theta = 34.2^\circ$ east of south.

EVALUATE: A graphical sketch confirms that this answer is reasonable.

- 1.86. IDENTIFY:** If the vector from your tent to Joe's is \vec{A} and from your tent to Karl's is \vec{B} , then the vector from Joe's tent to Karl's is $\vec{B} - \vec{A}$.

SET UP: Take your tent's position as the origin. Let $+x$ be east and $+y$ be north.

EXECUTE: The position vector for Joe's tent is

$$([21.0 \text{ m}]\cos 23^\circ)\hat{i} - ([21.0 \text{ m}]\sin 23^\circ)\hat{j} = (19.33 \text{ m})\hat{i} - (8.205 \text{ m})\hat{j}$$

The position vector for Karl's tent is $([32.0 \text{ m}]\cos 37^\circ)\hat{i} + ([32.0 \text{ m}]\sin 37^\circ)\hat{j} = (25.56 \text{ m})\hat{i} + (19.26 \text{ m})\hat{j}$.

The difference between the two positions is

$$(19.33 \text{ m} - 25.56 \text{ m})\hat{i} + (-8.205 \text{ m} - 19.26 \text{ m})\hat{j} = (-6.23 \text{ m})\hat{i} - (27.46 \text{ m})\hat{j}$$

The magnitude of this vector is the distance between the two tents: $D = \sqrt{(-6.23 \text{ m})^2 + (-27.46 \text{ m})^2} = 28.2 \text{ m}$

EVALUATE: If both tents were due east of yours, the distance between them would be $32.0 \text{ m} - 21.0 \text{ m} = 11.0 \text{ m}$. If Joe's was due north of yours and Karl's was due south of yours, then the distance between them would be $32.0 \text{ m} + 21.0 \text{ m} = 53.0 \text{ m}$. The actual distance between them lies between these limiting values.

- 1.87. IDENTIFY:** We know the scalar product and the magnitude of the vector product of two vectors and want to know the angle between them.

SET UP: The scalar product is $\vec{A} \cdot \vec{B} = AB \cos \theta$ and the vector product is $|\vec{A} \times \vec{B}| = AB \sin \theta$.

EXECUTE: $\vec{A} \cdot \vec{B} = AB \cos \theta = -6.00$ and $|\vec{A} \times \vec{B}| = AB \sin \theta = +9.00$. Taking the ratio gives $\tan \theta = \frac{9.00}{-6.00}$,
 so $\theta = 124^\circ$.

EVALUATE: Since the scalar product is negative, the angle must be between 90° and 180° .

- 1.88. IDENTIFY:** Calculate the scalar product and use Eq. (1.18) to determine ϕ .

SET UP: The unit vectors are perpendicular to each other.

EXECUTE: The direction vectors each have magnitude $\sqrt{3}$, and their scalar product is $(1)(1) + (1)(-1) + (1)(-1) = -1$, so from Eq. (1.18) the angle between the bonds is

$$\arccos\left(\frac{-1}{\sqrt{3}\sqrt{3}}\right) = \arccos\left(-\frac{1}{3}\right) = 109^\circ.$$

EVALUATE: The angle between the two vectors in the bond directions is greater than 90° .

- 1.89. IDENTIFY:** We know the magnitude of two vectors and their scalar product and want to find the magnitude of their vector product.

SET UP: The scalar product is $\vec{A} \cdot \vec{B} = AB \cos \theta$ and the vector product is $|\vec{A} \times \vec{B}| = AB \sin \theta$.

EXECUTE: $\vec{A} \cdot \vec{B} = AB \cos \theta = 90.0 \text{ m}^2$, which gives $\cos \theta = \frac{90.0 \text{ m}^2}{AB} = \frac{90.0 \text{ m}^2}{(12.0 \text{ m})(16.0 \text{ m})} = 0.4688$, so

$$\theta = 62.05^\circ. \text{ Therefore } |\vec{A} \times \vec{B}| = AB \sin \theta = (12.0 \text{ m})(16.0 \text{ m}) \sin 62.05^\circ = 170 \text{ m}^2.$$

EVALUATE: The magnitude of the vector product is greater than the scalar product because the angle between the vectors is greater than 45° .

- 1.90. IDENTIFY:** Let $\vec{C} = \vec{A} + \vec{B}$ and calculate the scalar product $\vec{C} \cdot \vec{C}$.

SET UP: For any vector \vec{V} , $\vec{V} \cdot \vec{V} = V^2$. $\vec{A} \cdot \vec{B} = AB \cos \phi$.

EXECUTE: (a) Use the linearity of the dot product to show that the square of the magnitude of the sum $\vec{A} + \vec{B}$ is

$$\begin{aligned} (\vec{A} + \vec{B}) \cdot (\vec{A} + \vec{B}) &= \vec{A} \cdot \vec{A} + \vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{A} + \vec{B} \cdot \vec{B} = \vec{A} \cdot \vec{A} + \vec{B} \cdot \vec{B} + 2\vec{A} \cdot \vec{B} = A^2 + B^2 + 2\vec{A} \cdot \vec{B} \\ &= A^2 + B^2 + 2AB \cos \phi \end{aligned}$$

(b) Using the result of part (a), with $A = B$, the condition is that $A^2 = A^2 + A^2 + 2A^2 \cos \phi$, which solves for $1 = 2 + 2 \cos \phi$, $\cos \phi = -\frac{1}{2}$, and $\phi = 120^\circ$.

EVALUATE: The expression $C^2 = A^2 + B^2 + 2AB \cos \phi$ is called the law of cosines.

- 1.91. IDENTIFY:** Find the angle between specified pairs of vectors.

SET UP: Use $\cos \phi = \frac{\vec{A} \cdot \vec{B}}{AB}$

EXECUTE: (a) $\vec{A} = \hat{k}$ (along line ab)

$\vec{B} = \hat{i} + \hat{j} + \hat{k}$ (along line ad)

$$A = 1, B = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

$$\vec{A} \cdot \vec{B} = \hat{k} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$$

$$\text{So } \cos \phi = \frac{\vec{A} \cdot \vec{B}}{AB} = 1/\sqrt{3}; \phi = 54.7^\circ$$

(b) $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ (along line ad)

$\vec{B} = \hat{j} + \hat{k}$ (along line ac)

$$A = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}; B = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\vec{A} \cdot \vec{B} = (\hat{i} + \hat{j} + \hat{k}) \cdot (\hat{j} + \hat{k}) = 1 + 1 = 2$$

$$\text{So } \cos \phi = \frac{\vec{A} \cdot \vec{B}}{AB} = \frac{2}{\sqrt{3}\sqrt{2}} = \frac{2}{\sqrt{6}}; \phi = 35.3^\circ$$

EVALUATE: Each angle is computed to be less than 90° , in agreement with what is deduced from Figure P1.91 in the textbook.

- 1.92. IDENTIFY:** We know the magnitude of two vectors and the magnitude of their vector product, and we want to find the possible values of their scalar product.

SET UP: The vector product is $|\vec{A} \times \vec{B}| = AB \sin \theta$ and the scalar product is $\vec{A} \cdot \vec{B} = AB \cos \theta$.

EXECUTE: $|\vec{A} \times \vec{B}| = AB \sin \theta = 12.0 \text{ m}^2$, so $\sin \theta = \frac{12.0 \text{ m}^2}{(6.00 \text{ m})(3.00 \text{ m})} = 0.6667$, which gives two possible

values: $\theta = 41.81^\circ$ or $\theta = 138.19^\circ$. Therefore the two possible values of the scalar product are

$$\vec{A} \cdot \vec{B} = AB \cos \theta = 13.4 \text{ m}^2 \text{ or } -13.4 \text{ m}^2.$$

EVALUATE: The two possibilities have equal magnitude but opposite sign because the two possible angles are supplementary to each other. The sines of these angles are the same but the cosines differ by a factor of -1 . See Figure 1.92.

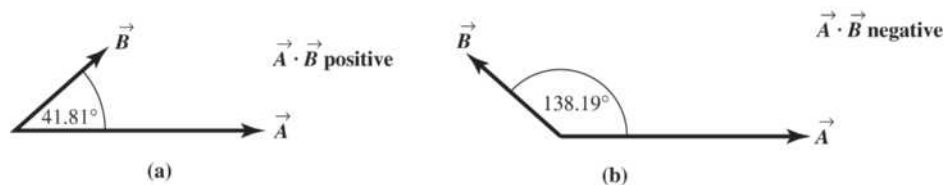


Figure 1.92

- 1.93. IDENTIFY:** We know the scalar product of two vectors, both their directions, and the magnitude of one of them, and we want to find the magnitude of the other vector.

SET UP: $\vec{A} \cdot \vec{B} = AB \cos \theta$. Since we know the direction of each vector, we can find the angle between them.

EXECUTE: The angle between the vectors is $\theta = 79.0^\circ$. Since $\vec{A} \cdot \vec{B} = AB \cos \theta$, we have

$$B = \frac{\vec{A} \cdot \vec{B}}{A \cos \theta} = \frac{48.0 \text{ m}^2}{(9.00 \text{ m}) \cos 79.0^\circ} = 28.0 \text{ m}.$$

EVALUATE: Vector \vec{B} has the same units as vector \vec{A} .

- 1.94. IDENTIFY:** The cross product $\vec{A} \times \vec{B}$ is perpendicular to both \vec{A} and \vec{B} .

SET UP: Use Eq. (1.27) to calculate the components of $\vec{A} \times \vec{B}$.

EXECUTE: The cross product is

$$(-13.00)\hat{i} + (6.00)\hat{j} + (-11.00)\hat{k} = 13 \left[-(1.00)\hat{i} + \left(\frac{6.00}{13.00}\right)\hat{j} - \frac{11.00}{13.00}\hat{k} \right].$$
 The magnitude of the vector in

square brackets is $\sqrt{1.93}$, and so a unit vector in this direction is

$$\left[\frac{-(1.00)\hat{i} + (6.00/13.00)\hat{j} - (11.00/13.00)\hat{k}}{\sqrt{1.93}} \right].$$

The negative of this vector,

$$\left[\frac{(1.00)\hat{i} - (6.00/13.00)\hat{j} + (11.00/13.00)\hat{k}}{\sqrt{1.93}} \right],$$

is also a unit vector perpendicular to \vec{A} and \vec{B} .

EVALUATE: Any two vectors that are not parallel or antiparallel form a plane and a vector perpendicular to both vectors is perpendicular to this plane.

- 1.95. IDENTIFY and SET UP:** The target variables are the components of \vec{C} . We are given \vec{A} and \vec{B} . We also know $\vec{A} \cdot \vec{C}$ and $\vec{B} \cdot \vec{C}$, and this gives us two equations in the two unknowns C_x and C_y .

EXECUTE: \vec{A} and \vec{C} are perpendicular, so $\vec{A} \cdot \vec{C} = 0$. $A_x C_x + A_y C_y = 0$, which gives $5.0 C_x - 6.5 C_y = 0$.

$$\vec{B} \cdot \vec{C} = 15.0, \text{ so } -3.5 C_x + 7.0 C_y = 15.0$$

We have two equations in two unknowns C_x and C_y . Solving gives $C_x = 8.0$ and $C_y = 6.1$.

EVALUATE: We can check that our result does give us a vector \vec{C} that satisfies the two equations $\vec{A} \cdot \vec{C} = 0$ and $\vec{B} \cdot \vec{C} = 15.0$.

1.96. IDENTIFY: Calculate the magnitude of the vector product and then use Eq. (1.22).

SET UP: The magnitude of a vector is related to its components by Eq. (1.12).

EXECUTE: $|\vec{A} \times \vec{B}| = AB \sin \theta$. $\sin \theta = \frac{|\vec{A} \times \vec{B}|}{AB} = \frac{\sqrt{(-5.00)^2 + (2.00)^2}}{(3.00)(3.00)} = 0.5984$ and

$$\theta = \sin^{-1}(0.5984) = 36.8^\circ.$$

EVALUATE: We haven't found \vec{A} and \vec{B} , just the angle between them.

1.97. (a) IDENTIFY: Prove that $\vec{A} \cdot (\vec{B} \times \vec{C}) = (\vec{A} \times \vec{B}) \cdot \vec{C}$.

SET UP: Express the scalar and vector products in terms of components.

EXECUTE:

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = A_x(\vec{B} \times \vec{C})_x + A_y(\vec{B} \times \vec{C})_y + A_z(\vec{B} \times \vec{C})_z$$

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = A_x(B_y C_z - B_z C_y) + A_y(B_z C_x - B_x C_z) + A_z(B_x C_y - B_y C_x)$$

$$(\vec{A} \times \vec{B}) \cdot \vec{C} = (\vec{A} \times \vec{B})_x C_x + (\vec{A} \times \vec{B})_y C_y + (\vec{A} \times \vec{B})_z C_z$$

$$(\vec{A} \times \vec{B}) \cdot \vec{C} = (A_y B_z - A_z B_y) C_x + (A_z B_x - A_x B_z) C_y + (A_x B_y - A_y B_x) C_z$$

Comparison of the expressions for $\vec{A} \cdot (\vec{B} \times \vec{C})$ and $(\vec{A} \times \vec{B}) \cdot \vec{C}$ shows they contain the same terms, so

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = (\vec{A} \times \vec{B}) \cdot \vec{C}.$$

(b) IDENTIFY: Calculate $(\vec{A} \times \vec{B}) \cdot \vec{C}$, given the magnitude and direction of \vec{A} , \vec{B} and \vec{C} .

SET UP: Use Eq. (1.22) to find the magnitude and direction of $\vec{A} \times \vec{B}$. Then we know the components of $\vec{A} \times \vec{B}$ and of \vec{C} and can use an expression like Eq. (1.21) to find the scalar product in terms of components.

EXECUTE: $A = 5.00$; $\theta_A = 26.0^\circ$; $B = 4.00$, $\theta_B = 63.0^\circ$

$$|\vec{A} \times \vec{B}| = AB \sin \phi.$$

The angle ϕ between \vec{A} and \vec{B} is equal to $\phi = \theta_B - \theta_A = 63.0^\circ - 26.0^\circ = 37.0^\circ$. So

$$|\vec{A} \times \vec{B}| = (5.00)(4.00) \sin 37.0^\circ = 12.04, \text{ and by the right hand-rule } \vec{A} \times \vec{B} \text{ is in the } +z\text{-direction. Thus}$$

$$(\vec{A} \times \vec{B}) \cdot \vec{C} = (12.04)(6.00) = 72.2$$

EVALUATE: $\vec{A} \times \vec{B}$ is a vector, so taking its scalar product with \vec{C} is a legitimate vector operation.

$(\vec{A} \times \vec{B}) \cdot \vec{C}$ is a scalar product between two vectors so the result is a scalar.

1.98. IDENTIFY: Use the maximum and minimum values of the dimensions to find the maximum and minimum areas and volumes.

SET UP: For a rectangle of width W and length L the area is LW . For a rectangular solid with dimensions L , W and H the volume is LWH .

EXECUTE: (a) The maximum and minimum areas are $(L+l)(W+w) = LW + lW + Lw$,

$(L-l)(W-w) = LW - lW - Lw$, where the common terms wl have been omitted. The area and its uncertainty are then $WL \pm (lW + Lw)$, so the uncertainty in the area is $a = lW + Lw$.

(b) The fractional uncertainty in the area is $\frac{a}{A} = \frac{lW + Lw}{WL} = \frac{l}{L} + \frac{w}{W}$, the sum of the fractional uncertainties in the length and width.

(c) The similar calculation to find the uncertainty v in the volume will involve neglecting the terms lwh , lWh and Lwh as well as lwh ; the uncertainty in the volume is $v = lWH + LwH + LW h$, and the fractional

uncertainty in the volume is $\frac{v}{V} = \frac{LWH + LwH + LW h}{LWH} = \frac{L}{L} + \frac{w}{W} + \frac{h}{H}$, the sum of the fractional uncertainties in the length, width and height.

EVALUATE: The calculation assumes the uncertainties are small, so that terms involving products of two or more uncertainties can be neglected.

- 1.99. IDENTIFY:** Add the vector displacements of the receiver and then find the vector from the quarterback to the receiver.

SET UP: Add the x -components and the y -components.

EXECUTE: The receiver's position is

$$[(+1.0 + 9.0 - 6.0 + 12.0)\text{yd}]\hat{i} + [(-5.0 + 11.0 + 4.0 + 18.0)\text{yd}]\hat{j} = (16.0\text{ yd})\hat{i} + (28.0\text{ yd})\hat{j}.$$

The vector from the quarterback to the receiver is the receiver's position minus the quarterback's position, or $(16.0\text{ yd})\hat{i} + (35.0\text{ yd})\hat{j}$, a vector with magnitude $\sqrt{(16.0\text{ yd})^2 + (35.0\text{ yd})^2} = 38.5\text{ yd}$. The angle is $\arctan\left(\frac{16.0}{35.0}\right) = 24.6^\circ$ to the right of downfield.

EVALUATE: The vector from the quarterback to receiver has positive x -component and positive y -component.

- 1.100. IDENTIFY:** Use the x and y coordinates for each object to find the vector from one object to the other; the distance between two objects is the magnitude of this vector. Use the scalar product to find the angle between two vectors.

SET UP: If object A has coordinates (x_A, y_A) and object B has coordinates (x_B, y_B) , the vector \vec{r}_{AB} from A to B has x -component $x_B - x_A$ and y -component $y_B - y_A$.

EXECUTE: (a) The diagram is sketched in Figure 1.100.

(b) (i) In AU, $\sqrt{(0.3182)^2 + (0.9329)^2} = 0.9857$.

(ii) In AU, $\sqrt{(1.3087)^2 + (-0.4423)^2 + (-0.0414)^2} = 1.3820$.

(iii) In AU $\sqrt{(0.3182 - 1.3087)^2 + (0.9329 - (-0.4423))^2 + (0.0414)^2} = 1.695$.

(c) The angle between the directions from the Earth to the Sun and to Mars is obtained from the dot product. Combining Eqs. (1.18) and (1.21),

$$\phi = \arccos\left(\frac{(-0.3182)(1.3087 - 0.3182) + (-0.9329)(-0.4423 - 0.9329) + (0)}{(0.9857)(1.695)}\right) = 54.6^\circ.$$

(d) Mars could not have been visible at midnight, because the Sun-Mars angle is less than 90° .

EVALUATE: Our calculations correctly give that Mars is farther from the Sun than the earth is. Note that on this date Mars was farther from the earth than it is from the Sun.

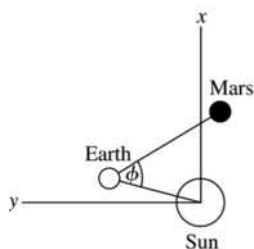


Figure 1.100

- 1.101. IDENTIFY:** Draw the vector addition diagram for the position vectors.

SET UP: Use coordinates in which the Sun to Merak line lies along the x -axis. Let \vec{A} be the position vector of Alkaid relative to the Sun, \vec{M} is the position vector of Merak relative to the Sun, and \vec{R} is the position vector for Alkaid relative to Merak. $A = 138\text{ ly}$ and $M = 77\text{ ly}$.

EXECUTE: The relative positions are shown in Figure 1.101. $\vec{M} + \vec{R} = \vec{A}$. $A_x = M_x + R_x$ so
 $R_x = A_x - M_x = (138 \text{ ly}) \cos 25.6^\circ - 77 \text{ ly} = 47.5 \text{ ly}$. $R_y = A_y - M_y = (138 \text{ ly}) \sin 25.6^\circ - 0 = 59.6 \text{ ly}$.
 $R = 76.2 \text{ ly}$ is the distance between Alkaid and Merak.

(b) The angle is angle ϕ in Figure 1.101. $\cos \theta = \frac{R_x}{R} = \frac{47.5 \text{ ly}}{76.2 \text{ ly}}$ and $\theta = 51.4^\circ$. Then $\phi = 180^\circ - \theta = 129^\circ$.

EVALUATE: The concepts of vector addition and components make these calculations very simple.

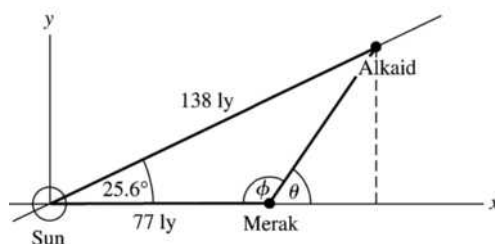


Figure 1.101

1.102. IDENTIFY: Define $\vec{S} = A\hat{i} + B\hat{j} + C\hat{k}$. Show that $\vec{r} \cdot \vec{S} = 0$ if $Ax + By + Cz = 0$.

SET UP: Use Eq. (1.21) to calculate the scalar product.

EXECUTE: $\vec{r} \cdot \vec{S} = (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (A\hat{i} + B\hat{j} + C\hat{k}) = Ax + By + Cz$

If the points satisfy $Ax + By + Cz = 0$, then $\vec{r} \cdot \vec{S} = 0$ and all points \vec{r} are perpendicular to \vec{S} . The vector and plane are sketched in Figure 1.102.

EVALUATE: If two vectors are perpendicular their scalar product is zero.

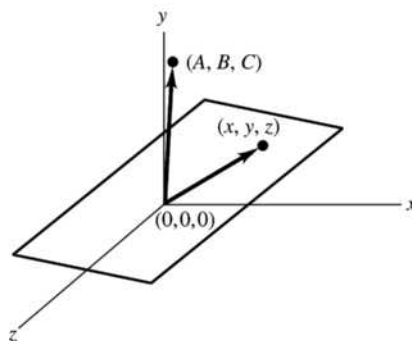


Figure 1.102

