9

ROTATION OF RIGID BODIES

9.1. **IDENTIFY:** $s = r\theta$, with θ in radians.

SET UP: $\pi \text{ rad} = 180°$.

EXECUTE: **(a)** $\theta = \dfrac{s}{r} = \dfrac{1.50\text{ m}}{2.50\text{ m}} = 0.600\text{ rad} = 34.4°$

(b) $r = \dfrac{s}{\theta} = \dfrac{14.0\text{ cm}}{(128°)(\pi\text{ rad}/180°)} = 6.27\text{ cm}$

(c) $s = r\theta = (1.50\text{ m})(0.700\text{ rad}) = 1.05\text{ m}$

EVALUATE: An angle is the ratio of two lengths and is dimensionless. But, when $s = r\theta$ is used, θ must be in radians. Or, if $\theta = s/r$ is used to calculate θ, the calculation gives θ in radians.

9.2. **IDENTIFY:** $\theta - \theta_0 = \omega t$, since the angular velocity is constant.

SET UP: $1\text{ rpm} = (2\pi/60)\text{ rad/s}$.

EXECUTE: **(a)** $\omega = (1900)(2\pi\text{ rad}/60\text{ s}) = 199\text{ rad/s}$

(b) $35° = (35°)(\pi/180°) = 0.611\text{ rad}$. $t = \dfrac{\theta - \theta_0}{\omega} = \dfrac{0.611\text{ rad}}{199\text{ rad/s}} = 3.1\times 10^{-3}\text{ s}$

EVALUATE: In $t = \dfrac{\theta - \theta_0}{\omega}$ we must use the same angular measure (radians, degrees or revolutions) for both $\theta - \theta_0$ and ω.

9.3. **IDENTIFY** $\alpha_z(t) = \dfrac{d\omega_z}{dt}$. Writing Eq. (2.16) in terms of angular quantities gives $\theta - \theta_0 = \int_{t_1}^{t_2} \omega_z dt$.

SET UP: $\dfrac{d}{dt}t^n = nt^{n-1}$ and $\int t^n dt = \dfrac{1}{n+1}t^{n+1}$

EXECUTE: **(a)** A must have units of rad/s and B must have units of rad/s^3.

(b) $\alpha_z(t) = 2Bt = (3.00\text{ rad/s}^3)t$. (i) For $t = 0$, $\alpha_z = 0$. (ii) For $t = 5.00\text{ s}$, $\alpha_z = 15.0\text{ rad/s}^2$.

(c) $\theta_2 - \theta_1 = \int_{t_1}^{t_2}(A + Bt^2)dt = A(t_2 - t_1) + \frac{1}{3}B(t_2^3 - t_1^3)$. For $t_1 = 0$ and $t_2 = 2.00\text{ s}$,

$\theta_2 - \theta_1 = (2.75\text{ rad/s})(2.00\text{ s}) + \frac{1}{3}(1.50\text{ rad/s}^3)(2.00\text{ s})^3 = 9.50\text{ rad}$.

EVALUATE: Both α_z and ω_z are positive and the angular speed is increasing.

9.4. **IDENTIFY:** $\alpha_z = d\omega_z/dt$. $\alpha_{\text{av-}z} = \dfrac{\Delta\omega_z}{\Delta t}$.

SET UP: $\dfrac{d}{dt}(t^2) = 2t$

EXECUTE: **(a)** $\alpha_z(t) = \dfrac{d\omega_z}{dt} = -2\beta t = (-1.60\text{ rad/s}^3)t$.

(b) $\alpha_z(3.0\text{ s}) = (-1.60\text{ rad/s}^3)(3.0\text{ s}) = -4.80\text{ rad/s}^2$.

$$\alpha_{\text{av-}z} = \frac{\omega_z(3.0\ \text{s}) - \omega_z(0)}{3.0\ \text{s}} = \frac{-2.20\ \text{rad/s} - 5.00\ \text{rad/s}}{3.0\ \text{s}} = -2.40\ \text{rad/s}^2,$$

which is half as large (in magnitude) as the acceleration at $t = 3.0$ s.

EVALUATE: $\alpha_z(t)$ increases linearly with time, so $\alpha_{\text{av-}z} = \frac{\alpha_z(0) + \alpha_z(3.0\ \text{s})}{2}$. $\alpha_z(0) = 0$.

9.5. **IDENTIFY** and **SET UP:** Use Eq. (9.3) to calculate the angular velocity and Eq. (9.2) to calculate the average angular velocity for the specified time interval.

EXECUTE: $\theta = \gamma t + \beta t^3$; $\gamma = 0.400$ rad/s, $\beta = 0.0120$ rad/s^3

(a) $\omega_z = \frac{d\theta}{dt} = \gamma + 3\beta t^2$

(b) At $t = 0$, $\omega_z = \gamma = 0.400$ rad/s

(c) At $t = 5.00$ s, $\omega_z = 0.400\ \text{rad/s} + 3(0.0120\ \text{rad/s}^3)(5.00\ \text{s})^2 = 1.30$ rad/s

$$\omega_{\text{av-}z} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$$

For $t_1 = 0$, $\theta_1 = 0$.

For $t_2 = 5.00$ s, $\theta_2 = (0.400\ \text{rad/s})(5.00\ \text{s}) + (0.012\ \text{rad/s}^3)(5.00\ \text{s})^3 = 3.50$ rad

So $\omega_{\text{av-}z} = \frac{3.50\ \text{rad} - 0}{5.00\ \text{s} - 0} = 0.700$ rad/s.

EVALUATE: The average of the instantaneous angular velocities at the beginning and end of the time interval is $\frac{1}{2}(0.400\ \text{rad/s} + 1.30\ \text{rad/s}) = 0.850$ rad/s. This is larger than $\omega_{\text{av-}z}$, because $\omega_z(t)$ is increasing faster than linearly.

9.6. **IDENTIFY:** $\omega_z(t) = \frac{d\theta}{dt}$. $\alpha_z(t) = \frac{d\omega_z}{dt}$. $\omega_{\text{av}-z} = \frac{\Delta\theta}{\Delta t}$.

SET UP: $\omega_z = (250\ \text{rad/s}) - (40.0\ \text{rad/s}^2)t - (4.50\ \text{rad/s}^3)t^2$. $\alpha_z = -(40.0\ \text{rad/s}^2) - (9.00\ \text{rad/s}^3)t$.

EXECUTE: **(a)** Setting $\omega_z = 0$ results in a quadratic in t. The only positive root is $t = 4.23$ s.

(b) At $t = 4.23$ s, $\alpha_z = -78.1$ rad/s^2.

(c) At $t = 4.23$ s, $\theta = 586\ \text{rad} = 93.3$ rev.

(d) At $t = 0$, $\omega_z = 250$ rad/s.

(e) $\omega_{\text{av-}z} = \frac{586\ \text{rad}}{4.23\ \text{s}} = 138$ rad/s.

EVALUATE: Between $t = 0$ and $t = 4.23$ s, ω_z decreases from 250 rad/s to zero. ω_z is not linear in t, so $\omega_{\text{av-}z}$ is not midway between the values of ω_z at the beginning and end of the interval.

9.7. **IDENTIFY:** $\omega_z(t) = \frac{d\theta}{dt}$. $\alpha_z(t) = \frac{d\omega_z}{dt}$. Use the values of θ and ω_z at $t = 0$ and α_z at 1.50 s to calculate a, b, and c.

SET UP: $\frac{d}{dt}t^n = nt^{n-1}$

EXECUTE: **(a)** $\omega_z(t) = b - 3ct^2$. $\alpha_z(t) = -6ct$. At $t = 0$, $\theta = a = \pi/4$ rad and $\omega_z = b = 2.00$ rad/s. At $t = 1.50$ s, $\alpha_z = -6c(1.50\ \text{s}) = 1.25$ rad/s^2 and $c = -0.139$ rad/s^3.

(b) $\theta = \pi/4$ rad and $\alpha_z = 0$ at $t = 0$.

(c) $\alpha_z = 3.50$ rad/s^2 at $t = -\frac{\alpha_z}{6c} = -\frac{3.50\ \text{rad/s}^2}{6(-0.139\ \text{rad/s}^3)} = 4.20$ s. At $t = 4.20$ s,

$$\theta = \frac{\pi}{4}\ \text{rad} + (2.00\ \text{rad/s})(4.20\ \text{s}) - (-0.139\ \text{rad/s}^3)(4.20\ \text{s})^3 = 19.5\ \text{rad}.$$

$\omega_z = 2.00 \text{ rad/s} - 3(-0.139 \text{ rad/s}^3)(4.20 \text{ s})^2 = 9.36 \text{ rad/s}.$

EVALUATE: θ, ω_z and α_z all increase as t increases.

9.8. **IDENTIFY:** $\alpha_z = \dfrac{d\omega_z}{dt}$. $\theta - \theta_0 = \omega_{\text{av-}z}t$. When ω_z is linear in t, $\omega_{\text{av-}z}$ for the time interval t_1 to t_2 is

$\omega_{\text{av-}z} = \dfrac{\omega_{z1} + \omega_{z2}}{t_2 - t_1}$.

SET UP: From the information given, $\omega_z(t) = -6.00 \text{ rad/s} + (2.00 \text{ rad/s}^2)t$.

EXECUTE: **(a)** The angular acceleration is positive, since the angular velocity increases steadily from a negative value to a positive value.

(b) It takes 3.00 seconds for the wheel to stop $(\omega_z = 0)$. During this time its speed is decreasing. For the next 4.00 s its speed is increasing from 0 rad/s to $+8.00 \text{ rad/s}$.

(c) The average angular velocity is $\dfrac{-6.00 \text{ rad/s} + 8.00 \text{ rad/s}}{2} = 1.00 \text{ rad/s}$. $\theta - \theta_0 = \omega_{\text{av-}z}t$ then leads to displacement of 7.00 rad after 7.00 s.

EVALUATE: When α_z and ω_z have the same sign, the angular speed is increasing; this is the case for $t = 3.00 \text{ s}$ to $t = 7.00 \text{ s}$. When α_z and ω_z have opposite signs, the angular speed is decreasing; this is the case between $t = 0$ and $t = 3.00 \text{ s}$.

9.9. **IDENTIFY:** Apply the constant angular acceleration equations.

SET UP: Let the direction the wheel is rotating be positive.

EXECUTE: **(a)** $\omega_z = \omega_{0z} + \alpha_z t = 1.50 \text{ rad/s} + (0.300 \text{ rad/s}^2)(2.50 \text{ s}) = 2.25 \text{ rad/s}.$

(b) $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2 = (1.50 \text{ rad/s})(2.50 \text{ s}) + \frac{1}{2}(0.300 \text{ rad/s}^2)(2.50 \text{ s})^2 = 4.69 \text{ rad}.$

EVALUATE: $\theta - \theta_0 = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t = \left(\dfrac{1.50 \text{ rad/s} + 2.25 \text{ rad/s}}{2}\right)(2.50 \text{ s}) = 4.69 \text{ rad}$, the same as calculated with another equation in part (b).

9.10. **IDENTIFY:** Apply the constant angular acceleration equations to the motion of the fan.

(a) SET UP: $\omega_{0z} = (500 \text{ rev/min})(1 \text{ min}/60 \text{ s}) = 8.333 \text{ rev/s}$, $\omega_z = (200 \text{ rev/min})(1 \text{ min}/60 \text{ s}) = 3.333 \text{ rev/s}$, $t = 4.00 \text{ s}$, $\alpha_z = ?$

$\omega_z = \omega_{0z} + \alpha_z t$

EXECUTE: $\alpha_z = \dfrac{\omega_z - \omega_{0z}}{t} = \dfrac{3.333 \text{ rev/s} - 8.333 \text{ rev/s}}{4.00 \text{ s}} = -1.25 \text{ rev/s}^2$

$\theta - \theta_0 = ?$

$\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2 = (8.333 \text{ rev/s})(4.00 \text{ s}) + \frac{1}{2}(-1.25 \text{ rev/s}^2)(4.00 \text{ s})^2 = 23.3 \text{ rev}$

(b) SET UP: $\omega_z = 0$ (comes to rest); $\omega_{0z} = 3.333 \text{ rev/s}$; $\alpha_z = -1.25 \text{ rev/s}^2$;

$t = ?$

$\omega_z = \omega_{0z} + \alpha_z t$

EXECUTE: $t = \dfrac{\omega_z - \omega_{0z}}{\alpha_z} = \dfrac{0 - 3.333 \text{ rev/s}}{-1.25 \text{ rev/s}^2} = 2.67 \text{ s}$

EVALUATE: The angular acceleration is negative because the angular velocity is decreasing. The average angular velocity during the 4.00 s time interval is 350 rev/min and $\theta - \theta_0 = \omega_{\text{av-}z}t$ gives $\theta - \theta_0 = 23.3 \text{ rev}$, which checks.

9.11. **IDENTIFY:** Apply the constant angular acceleration equations to the motion. The target variables are t and $\theta - \theta_0$.

SET UP: **(a)** $\alpha_z = 1.50 \text{ rad/s}^2$; $\omega_{0z} = 0$ (starts from rest); $\omega_z = 36.0 \text{ rad/s}$; $t = ?$

$\omega_z = \omega_{0z} + \alpha_z t$

EXECUTE: $t = \dfrac{\omega_z - \omega_{0z}}{\alpha_z} = \dfrac{36.0 \text{ rad/s} - 0}{1.50 \text{ rad/s}^2} = 24.0 \text{ s}$

(b) $\theta - \theta_0 = ?$

$\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2 = 0 + \frac{1}{2}(1.50 \text{ rad/s}^2)(24.0 \text{ s})^2 = 432 \text{ rad}$

$\theta - \theta_0 = 432 \text{ rad}(1 \text{ rev}/2\pi \text{ rad}) = 68.8 \text{ rev}$

EVALUATE: We could use $\theta - \theta_0 = \frac{1}{2}(\omega_z + \omega_{0z})t$ to calculate $\theta - \theta_0 = \frac{1}{2}(0 + 36.0 \text{ rad/s})(24.0 \text{ s}) = 432 \text{ rad}$, which checks.

9.12. **IDENTIFY:** In part (b) apply the equation derived in part (a).

SET UP: Let the direction the propeller is rotating be positive.

EXECUTE: **(a)** Solving Eq. (9.7) for *t* gives $t = \dfrac{\omega_z - \omega_{0z}}{\alpha_z}$. Rewriting Eq. (9.11) as $\theta - \theta_0 = t(\omega_{0z} + \frac{1}{2}\alpha_z t)$ and substituting for *t* gives

$$\theta - \theta_0 = \left(\frac{\omega_z - \omega_{0z}}{\alpha_z}\right)\left(\omega_{0z} + \frac{1}{2}(\omega_z - \omega_{0z})\right) = \frac{1}{\alpha_z}(\omega_z - \omega_{0z})\left(\frac{\omega_z + \omega_{0z}}{2}\right) = \frac{1}{2\alpha_z}(\omega_z^2 - \omega_{0z}^2),$$

which when rearranged gives Eq. (9.12).

(b) $\alpha_z = \frac{1}{2}\left(\dfrac{1}{\theta - \theta_0}\right)\left(\omega_z^2 - \omega_{0z}^2\right) = \frac{1}{2}\left(\dfrac{1}{7.00 \text{ rad}}\right)\left((16.0 \text{ rad/s})^2 - (12.0 \text{ rad/s})^2\right) = 8.00 \text{ rad/s}^2$

EVALUATE: We could also use $\theta - \theta_0 = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t$ to calculate $t = 0.500$ s. Then $\omega_z = \omega_{0z} + \alpha_z t$ gives $\alpha_z = 8.00 \text{ rad/s}^2$, which agrees with our results in part (b).

9.13. **IDENTIFY:** Use a constant angular acceleration equation and solve for ω_{0z}.

SET UP: Let the direction of rotation of the flywheel be positive.

EXECUTE: $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2$ gives $\omega_{0z} = \dfrac{\theta - \theta_0}{t} - \frac{1}{2}a_z\, t = \dfrac{60.0 \text{ rad}}{4.00 \text{ s}} - \frac{1}{2}(2.25 \text{ rad/s}^2)(4.00 \text{ s}) = 10.5 \text{ rad/s}.$

EVALUATE: At the end of the 4.00 s interval, $\omega_z = \omega_{0z} + \alpha_z t = 19.5$ rad/s.

$\theta - \theta_0 = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t = \left(\dfrac{10.5 \text{ rad/s} + 19.5 \text{ rad/s}}{2}\right)(4.00 \text{ s}) = 60.0 \text{ rad}$, which checks.

9.14. **IDENTIFY:** Apply the constant angular acceleration equations.

SET UP: Let the direction of the rotation of the blade be positive. $\omega_{0z} = 0$.

EXECUTE: $\omega_z = \omega_{0z} + \alpha_z t$ gives $\alpha_z = \dfrac{\omega_z - \omega_{0z}}{t} = \dfrac{140 \text{ rad/s} - 0}{6.00 \text{ s}} = 23.3 \text{ rad/s}^2.$

$(\theta - \theta_0) = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t = \left(\dfrac{0 + 140 \text{ rad/s}}{2}\right)(6.00 \text{ s}) = 420 \text{ rad}$

EVALUATE: We could also use $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2$. This equation gives

$\theta - \theta_0 = \frac{1}{2}(23.3 \text{ rad/s}^2)(6.00 \text{ s})^2 = 419 \text{ rad}$, in agreement with the result obtained above.

9.15. **IDENTIFY:** Apply constant angular acceleration equations.

SET UP: Let the direction the flywheel is rotating be positive.

$\theta - \theta_0 = 200 \text{ rev}, \omega_{0z} = 500 \text{ rev/min} = 8.333 \text{ rev/s}, t = 30.0 \text{ s}.$

EXECUTE: **(a)** $\theta - \theta_0 = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t$ gives $\omega_z = 5.00 \text{ rev/s} = 300 \text{ rpm}$

(b) Use the information in part (a) to find α_z: $\omega_z = \omega_{0z} + \alpha_z t$ gives $\alpha_z = -0.1111 \text{ rev/s}^2$. Then $\omega_z = 0$, $\alpha_z = -0.1111 \text{ rev/s}^2, \omega_{0z} = 8.333 \text{ rev/s}$ in $\omega_z = \omega_{0z} + \alpha_z t$ gives $t = 75.0$ s and $\theta - \theta_0 = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t$ gives $\theta - \theta_0 = 312$ rev.

EVALUATE: The mass and diameter of the flywheel are not used in the calculation.

9.16. **IDENTIFY:** Apply the constant angular acceleration equations separately to the time intervals 0 to 2.00 s and 2.00 s until the wheel stops.

(a) SET UP: Consider the motion from $t = 0$ to $t = 2.00$ s:

$\theta - \theta_0 = ?$; $\omega_{0z} = 24.0$ rad/s; $\alpha_z = 30.0\ \text{rad/s}^2$; $t = 2.00$ s

EXECUTE: $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2 = (24.0\ \text{rad/s})(2.00\ \text{s}) + \frac{1}{2}(30.0\ \text{rad/s}^2)(2.00\ \text{s})^2$

$\theta - \theta_0 = 48.0\ \text{rad} + 60.0\ \text{rad} = 108\ \text{rad}$

Total angular displacement from $t = 0$ until stops: $108\ \text{rad} + 432\ \text{rad} = 540\ \text{rad}$

Note: At $t = 2.00$ s, $\omega_z = \omega_{0z} + \alpha_z t = 24.0\ \text{rad/s} + (30.0\ \text{rad/s}^2)(2.00\ \text{s}) = 84.0\ \text{rad/s}$; angular speed when breaker trips.

(b) SET UP: Consider the motion from when the circuit breaker trips until the wheel stops. For this calculation let $t = 0$ when the breaker trips.

$t = ?$; $\theta - \theta_0 = 432$ rad; $\omega_z = 0$; $\omega_{0z} = 84.0$ rad/s (from part (a))

$$\theta - \theta_0 = \left(\frac{\omega_{0z} + \omega_z}{2}\right)t$$

EXECUTE: $t = \dfrac{2(\theta - \theta_0)}{\omega_{0z} + \omega_z} = \dfrac{2(432\ \text{rad})}{84.0\ \text{rad/s} + 0} = 10.3\ \text{s}$

The wheel stops 10.3 s after the breaker trips so $2.00\ \text{s} + 10.3\ \text{s} = 12.3\ \text{s}$ from the beginning.

(c) SET UP: $\alpha_z = ?$; consider the same motion as in part (b):

$\omega_z = \omega_{0z} + \alpha_z t$

EXECUTE: $\alpha_z = \dfrac{\omega_z - \omega_{0z}}{t} = \dfrac{0 - 84.0\ \text{rad/s}}{10.3\ \text{s}} = -8.16\ \text{rad/s}^2$

EVALUATE: The angular acceleration is positive while the wheel is speeding up and negative while it is slowing down. We could also use $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ to calculate

$\alpha_z = \dfrac{\omega_z^2 - \omega_{0z}^2}{2(\theta - \theta_0)} = \dfrac{0 - (84.0\ \text{rad/s})^2}{2(432\ \text{rad})} = -8.16\ \text{rad/s}^2$ for the acceleration after the breaker trips.

9.17. **IDENTIFY:** Apply Eq. (9.12) to relate ω_z to $\theta - \theta_0$.

SET UP: Establish a proportionality.

EXECUTE: From Eq. (9.12), with $\omega_{0z} = 0$, the number of revolutions is proportional to the square of the initial angular velocity, so tripling the initial angular velocity increases the number of revolutions by 9, to 9.00 rev.

EVALUATE: We don't have enough information to calculate α_z; all we need to know is that it is constant.

9.18. **IDENTIFY:** The linear distance the elevator travels, its speed and the magnitude of its acceleration are equal to the tangential displacement, speed and acceleration of a point on the rim of the disk. $s = r\theta$, $v = r\omega$ and $a = r\alpha$. In these equations the angular quantities must be in radians.

SET UP: $1\ \text{rev} = 2\pi$ rad. $1\ \text{rpm} = 0.1047$ rad/s. $\pi\ \text{rad} = 180°$. For the disk, $r = 1.25$ m.

EXECUTE: **(a)** $v = 0.250$ m/s so $\omega = \dfrac{v}{r} = \dfrac{0.250\ \text{m/s}}{1.25\ \text{m}} = 0.200\ \text{rad/s} = 1.91\ \text{rpm}$.

(b) $a = \frac{1}{8}g = 1.225\ \text{m/s}^2$. $\alpha = \dfrac{a}{r} = \dfrac{1.225\ \text{m/s}^2}{1.25\ \text{m}} = 0.980\ \text{rad/s}^2$.

(c) $s = 3.25$ m. $\theta = \dfrac{s}{r} = \dfrac{3.25\ \text{m}}{1.25\ \text{m}} = 2.60\ \text{rad} = 149°$.

EVALUATE: When we use $s = r\theta$, $v = r\omega$ and $a_{\tan} = r\alpha$ to solve for θ, ω and α, the results are in rad, rad/s and rad/s^2.

9.19. **IDENTIFY:** When the angular speed is constant, $\omega = \theta/t$. $v_{\tan} = r\omega$, $a_{\tan} = r\alpha$ and $a_{\text{rad}} = r\omega^2$. In these equations radians must be used for the angular quantities.

SET UP: The radius of the earth is $R_{\text{E}} = 6.38\times10^6$ m and the earth rotates once in 1 day $= 86{,}400$ s. The orbit radius of the earth is 1.50×10^{11} m and the earth completes one orbit in $1\text{ y} = 3.156\times10^7$ s. When ω is constant, $\omega = \theta/t$.

EXECUTE: **(a)** $\theta = 1\text{ rev} = 2\pi$ rad in $t = 3.156\times10^7$ s. $\omega = \dfrac{2\pi\text{ rad}}{3.156\times10^7\text{ s}} = 1.99\times10^{-7}$ rad/s.

(b) $\theta = 1\text{ rev} = 2\pi$ rad in $t = 86{,}400$ s. $\omega = \dfrac{2\pi\text{ rad}}{86{,}400\text{ s}} = 7.27\times10^{-5}$ rad/s

(c) $v = r\omega = (1.50\times10^{11}\text{ m})(1.99\times10^{-7}\text{ rad/s}) = 2.98\times10^4$ m/s.

(d) $v = r\omega = (6.38\times10^6\text{ m})(7.27\times10^{-5}\text{ rad/s}) = 464$ m/s.

(e) $a_{\text{rad}} = r\omega^2 = (6.38\times10^6\text{ m})(7.27\times10^{-5}\text{ rad/s})^2 = 0.0337\text{ m/s}^2$. $a_{\tan} = r\alpha = 0$. $\alpha = 0$ since the angular velocity is constant.

EVALUATE: The tangential speeds associated with these motions are large even though the angular speeds are very small, because the radius for the circular path in each case is quite large.

9.20. **IDENTIFY:** Linear and angular velocities are related by $v = r\omega$. Use $\omega_z = \omega_{0z} + \alpha_z t$ to calculate α_z.

SET UP: $\omega = v/r$ gives ω in rad/s.

EXECUTE: **(a)** $\dfrac{1.25\text{ m/s}}{25.0\times10^{-3}\text{ m}} = 50.0$ rad/s, $\dfrac{1.25\text{ m/s}}{58.0\times10^{-3}\text{ m}} = 21.6$ rad/s.

(b) $(1.25\text{ m/s})(74.0\text{ min})(60\text{ s/min}) = 5.55$ km.

(c) $\alpha_z = \dfrac{21.55\text{ rad/s} - 50.0\text{ rad/s}}{(74.0\text{ min})(60\text{ s/min})} = -6.41\times10^{-3}\text{ rad/s}^2$.

EVALUATE: The width of the tracks is very small, so the total track length on the disc is huge.

9.21. **IDENTIFY:** Use constant acceleration equations to calculate the angular velocity at the end of two revolutions. $v = r\omega$.

SET UP: $2\text{ rev} = 4\pi$ rad. $r = 0.200$ m.

EXECUTE: **(a)** $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$. $\omega_z = \sqrt{2\alpha_z(\theta - \theta_0)} = \sqrt{2(3.00\text{ rad/s}^2)(4\pi\text{ rad})} = 8.68$ rad/s.

$a_{\text{rad}} = r\omega^2 = (0.200\text{ m})(8.68\text{ rad/s})^2 = 15.1\text{ m/s}^2$.

(b) $v = r\omega = (0.200\text{ m})(8.68\text{ rad/s}) = 1.74$ m/s. $a_{\text{rad}} = \dfrac{v^2}{r} = \dfrac{(1.74\text{ m/s})^2}{0.200\text{ m}} = 15.1\text{ m/s}^2$.

EVALUATE: $r\omega^2$ and v^2/r are completely equivalent expressions for a_{rad}.

9.22. **IDENTIFY:** $v = r\omega$ and $a_{\tan} = r\alpha$.

SET UP: The linear acceleration of the bucket equals $a_{\tan}$ for a point on the rim of the axle.

EXECUTE: **(a)** $v = R\omega$. $2.00\text{ cm/s} = R\left(\dfrac{7.5\text{ rev}}{\text{min}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right)\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right)$ gives $R = 2.55$ cm.

$D = 2R = 5.09$ cm.

(b) $a_{\tan} = R\alpha$. $\alpha = \dfrac{a_{\tan}}{R} = \dfrac{0.400\text{ m/s}^2}{0.0255\text{ m}} = 15.7\text{ rad/s}^2$.

EVALUATE: In $v = R\omega$ and $a_{\tan} = R\alpha$, ω and α must be in radians.

9.23. **IDENTIFY** and **SET UP:** Use constant acceleration equations to find ω and α after each displacement. Then use Eqs. (9.14) and (9.15) to find the components of the linear acceleration.

EXECUTE: **(a)** at the start $t = 0$

flywheel starts from rest so $\omega = \omega_{0z} = 0$

$a_{\tan} = r\alpha = (0.300\text{ m})(0.600\text{ rad/s}^2) = 0.180\text{ m/s}^2$

$a_{\text{rad}} = r\omega^2 = 0$

$a = \sqrt{a_{\text{rad}}^2 + a_{\tan}^2} = 0.180\ \text{m/s}^2$

(b) $\underline{\theta - \theta_0 = 60°}$

$a_{\tan} = r\alpha = 0.180\ \text{m/s}^2$

Calculate ω:

$\theta - \theta_0 = 60°(\pi\ \text{rad}/180°) = 1.047\ \text{rad};\ \ \omega_{0z} = 0;\ \ \alpha_z = 0.600\ \text{rad/s}^2;\ \ \omega_z = ?$

$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$

$\omega_z = \sqrt{2\alpha_z(\theta - \theta_0)} = \sqrt{2(0.600\ \text{rad/s}^2)(1.047\ \text{rad})} = 1.121\ \text{rad/s}$ and $\omega = \omega_z$.

Then $a_{\text{rad}} = r\omega^2 = (0.300\ \text{m})(1.121\ \text{rad/s})^2 = 0.377\ \text{m/s}^2$.

$a = \sqrt{a_{\text{rad}}^2 + a_{\tan}^2} = \sqrt{(0.377\ \text{m/s}^2)^2 + (0.180\ \text{m/s}^2)^2} = 0.418\ \text{m/s}^2$

(c) $\underline{\theta - \theta_0 = 120°}$

$a_{\tan} = r\alpha = 0.180\ \text{m/s}^2$

Calculate ω:

$\theta - \theta_0 = 120°(\pi\ \text{rad}/180°) = 2.094\ \text{rad};\ \ \omega_{0z} = 0;\ \ \alpha_z = 0.600\ \text{rad/s}^2;\ \ \omega_z = ?$

$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$

$\omega_z = \sqrt{2\alpha_z(\theta - \theta_0)} = \sqrt{2(0.600\ \text{rad/s}^2)(2.094\ \text{rad})} = 1.585\ \text{rad/s}$ and $\omega = \omega_z$.

Then $a_{\text{rad}} = r\omega^2 = (0.300\ \text{m})(1.585\ \text{rad/s})^2 = 0.754\ \text{m/s}^2$.

$a = \sqrt{a_{\text{rad}}^2 + a_{\tan}^2} = \sqrt{(0.754\ \text{m/s}^2)^2 + (0.180\ \text{m/s}^2)^2} = 0.775\ \text{m/s}^2$.

EVALUATE: α is constant so $\alpha_{\tan}$ is constant. ω increases so a_{rad} increases.

9.24. IDENTIFY: Apply constant angular acceleration equations. $v = r\omega$. A point on the rim has both tangential and radial components of acceleration.

SET UP: $a_{\tan} = r\alpha$ and $a_{\text{rad}} = r\omega^2$.

EXECUTE: **(a)** $\omega_z = \omega_{0z} + \alpha_z t\ =\ 0.250\ \text{rev/s} + (0.900\ \text{rev/s}^2)(0.200\ \text{s}) = 0.430\ \text{rev/s}$

(Note that since ω_{0z} and α_z are given in terms of revolutions, it's not necessary to convert to radians).

(b) $\omega_{\text{av-}z}\Delta t = (0.340\ \text{rev/s})(0.2\ \text{s}) = 0.068\ \text{rev}$.

(c) Here, the conversion to radians must be made to use Eq. (9.13), and

$$v = r\omega = \left(\frac{0.750\ \text{m}}{2}\right)(0.430\ \text{rev/s})(2\pi\ \text{rad/rev}) = 1.01\ \text{m/s}.$$

(d) Combining Eqs. (9.14) and (9.15),

$a = \sqrt{a_{\text{rad}}^2 + a_{\tan}^2} = \sqrt{(\omega^2 r)^2 + (\alpha r)^2}$.

$a = \sqrt{\left[((0.430\ \text{rev/s})(2\pi\ \text{rad/rev}))^2(0.375\ \text{m})\right]^2 + \left[(0.900\ \text{rev/s}^2)(2\pi\ \text{rad/rev})(0.375\ \text{m})\right]^2}$.

$a = 3.46\ \text{m/s}^2$.

EVALUATE: If the angular acceleration is constant, $a_{\tan}$ is constant but a_{rad} increases as ω increases.

9.25. IDENTIFY: Use Eq. (9.15) and solve for r.

SET UP: $a_{\text{rad}} = r\omega^2$ so $r = a_{\text{rad}}/\omega^2$, where ω must be in rad/s

EXECUTE: $a_{\text{rad}} = 3000g = 3000(9.80\ \text{m/s}^2) = 29{,}400\ \text{m/s}^2$

$\omega = (5000\ \text{rev/min})\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = 523.6\ \text{rad/s}$

Then $r = \dfrac{a_{\text{rad}}}{\omega^2} = \dfrac{29{,}400\text{ m/s}^2}{(523.6\text{ rad/s})^2} = 0.107\text{ m}.$

EVALUATE: The diameter is then 0.214 m, which is larger than 0.127 m, so the claim is *not* realistic.

9.26. **IDENTIFY:** In part (b) apply the result derived in part (a).

SET UP: $a_{\text{rad}} = r\omega^2$ and $v = r\omega;$ combine to eliminate r.

EXECUTE: **(a)** $a_{\text{rad}} = \omega^2 r = \omega^2\left(\dfrac{v}{\omega}\right) = \omega v.$

(b) From the result of part (a), $\omega = \dfrac{a_{\text{rad}}}{v} = \dfrac{0.500\text{ m/s}^2}{2.00\text{ m/s}} = 0.250\text{ rad/s}.$

EVALUATE: $a_{\text{rad}} = r\omega^2$ and $v = r\omega$ both require that ω be in rad/s, so in $a_{\text{rad}} = \omega v$, ω is in rad/s.

9.27. **IDENTIFY:** $v = r\omega$ and $a_{\text{rad}} = r\omega^2 = v^2/r.$

SET UP: 2π rad $= 1$ rev, so π rad/s $= 30$ rev/min.

EXECUTE: **(a)** $\omega r = (1250\text{ rev/min})\left(\dfrac{\pi\text{ rad/s}}{30\text{ rev/min}}\right)\left(\dfrac{12.7\times 10^{-3}\text{m}}{2}\right) = 0.831\text{ m/s}.$

(b) $\dfrac{v^2}{r} = \dfrac{(0.831\text{ m/s})^2}{(12.7\times 10^{-3}\text{ m})/2} = 109\text{ m/s}^2.$

EVALUATE: In $v = r\omega$, ω must be in rad/s.

9.28. **IDENTIFY:** $a_{\tan} = r\alpha$, $v = r\omega$ and $a_{\text{rad}} = v^2/r$. $\theta - \theta_0 = \omega_{\text{av-}z}t.$

SET UP: When α_z is constant, $\omega_{\text{av-}z} = \dfrac{\omega_{0z} + \omega_z}{2}$. Let the direction the wheel is rotating be positive.

EXECUTE: **(a)** $\alpha = \dfrac{a_{\tan}}{r} = \dfrac{-10.0\text{ m/s}^2}{0.200\text{ m}} = -50.0\text{ rad/s}^2$

(b) At $t = 3.00$ s, $v = 50.0$ m/s and $\omega = \dfrac{v}{r} = \dfrac{50.0\text{ m/s}}{0.200\text{ m}} = 250\text{ rad/s}$ and at $t = 0$,

$v = 50.0\text{ m/s} + (-10.0\text{ m/s}^2)(0 - 3.00\text{ s}) = 80.0\text{ m/s}$, $\omega = 400$ rad/s.

(c) $\omega_{\text{av-}z}t = (325\text{ rad/s})(3.00\text{ s}) = 975\text{ rad} = 155\text{ rev}.$

(d) $v = \sqrt{a_{\text{rad}}r} = \sqrt{(9.80\text{ m/s}^2)(0.200\text{ m})} = 1.40\text{ m/s}$. This speed will be reached at time $\dfrac{50.0\text{ m/s} - 1.40\text{ m/s}}{10.0\text{ m/s}^2} = 4.86\text{ s}$ after $t = 3.00$ s, or at $t = 7.86$ s. (There are many equivalent ways to do this calculation.)

EVALUATE: At $t = 0$, $a_{\text{rad}} = r\omega^2 = 3.20\times 10^4\text{ m/s}^2$. At $t = 3.00$ s, $a_{\text{rad}} = 1.25\times 10^4\text{ m/s}^2$. For $a_{\text{rad}} = g$ the wheel must be rotating more slowly than at 3.00 s so it occurs some time after 3.00 s.

9.29. **IDENTIFY** and **SET UP:** Use Eq. (9.15) to relate ω to a_{rad} and $\sum\vec{F} = m\vec{a}$ to relate a_{rad} to F_{rad}. Use Eq. (9.13) to relate ω and v, where v is the tangential speed.

EXECUTE: **(a)** $a_{\text{rad}} = r\omega^2$ and $F_{\text{rad}} = ma_{\text{rad}} = mr\omega^2$

$$\frac{F_{\text{rad},2}}{F_{\text{rad},1}} = \left(\frac{\omega_2}{\omega_1}\right)^2 = \left(\frac{640\text{ rev/min}}{423\text{ rev/min}}\right)^2 = 2.29$$

(b) $v = r\omega$

$$\frac{v_2}{v_1} = \frac{\omega_2}{\omega_1} = \frac{640\text{ rev/min}}{423\text{ rev/min}} = 1.51$$

(c) $v = r\omega$

$$\omega = (640\text{ rev/min})\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 67.0\text{ rad/s}$$

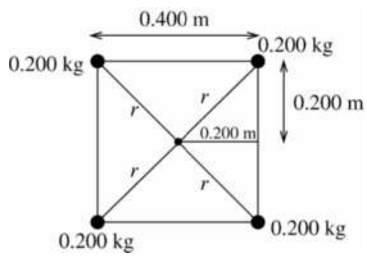
Then $v = r\omega = (0.235\text{ m})(67.0\text{ rad/s}) = 15.7\text{ m/s}$.

$a_{\text{rad}} = r\omega^2 = (0.235\text{ m})(67.0\text{ rad/s})^2 = 1060\text{ m/s}^2$

$\dfrac{a_{\text{rad}}}{g} = \dfrac{1060\text{ m/s}^2}{9.80\text{ m/s}^2} = 108;\quad a = 108g$

EVALUATE: In parts (a) and (b), since a ratio is used the units cancel and there is no need to convert ω to rad/s. In part (c), v and a_{rad} are calculated from ω, and ω must be in rad/s.

9.30. **IDENTIFY** and **SET UP:** Use Eq. (9.16). Treat the spheres as point masses and ignore I of the light rods.
EXECUTE: The object is shown in Figure 9.30a.
(a)



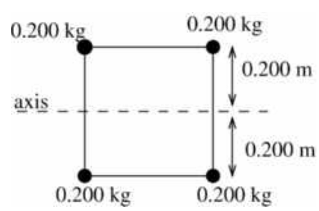
$r = \sqrt{(0.200\text{ m})^2 + (0.200\text{ m})^2} = 0.2828\text{ m}$

$I = \sum m_i r_i^2 = 4(0.200\text{ kg})(0.2828\text{ m})^2$

$I = 0.0640\text{ kg}\cdot\text{m}^2$

Figure 9.30a

(b) The object is shown in Figure 9.30b.



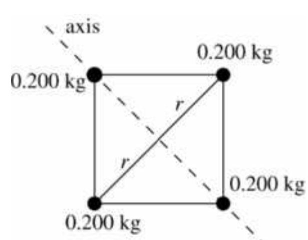
$r = 0.200\text{ m}$

$I = \sum m_i r_i^2 = 4(0.200\text{ kg})(0.200\text{ m})^2$

$I = 0.0320\text{ kg}\cdot\text{m}^2$

Figure 9.30b

(c) The object is shown in Figure 9.30c.



$r = 0.2828\text{ m}$

$I = \sum m_i r_i^2 = 2(0.200\text{ kg})(0.2828\text{ m})^2$

$I = 0.0320\text{ kg}\cdot\text{m}^2$

Figure 9.30c

EVALUATE: In general I depends on the axis and our answer for part (a) is larger than for parts (b) and (c). It just happens that I is the same in parts (b) and (c).

9.31. **IDENTIFY:** Use Table 9.2. The correct expression to use in each case depends on the shape of the object and the location of the axis.
SET UP: In each case express the mass in kg and the length in m, so the moment of inertia will be in $\text{kg}\cdot\text{m}^2$.
EXECUTE: **(a)** (i) $I = \frac{1}{3}ML^2 = \frac{1}{3}(2.50\text{ kg})(0.750\text{ m})^2 = 0.469\text{ kg}\cdot\text{m}^2$.

(ii) $I = \frac{1}{12}ML^2 = \frac{1}{4}(0.469\ \text{kg}\cdot\text{m}^2) = 0.117\ \text{kg}\cdot\text{m}^2$. (iii) For a very thin rod, all of the mass is at the axis and $I = 0$.

(b) (i) $I = \frac{2}{5}MR^2 = \frac{2}{5}(3.00\ \text{kg})(0.190\ \text{m})^2 = 0.0433\ \text{kg}\cdot\text{m}^2$.

(ii) $I = \frac{2}{3}MR^2 = \frac{5}{3}(0.0433\ \text{kg}\cdot\text{m}^2) = 0.0722\ \text{kg}\cdot\text{m}^2$.

(c) (i) $I = MR^2 = (8.00\ \text{kg})(0.0600\ \text{m})^2 = 0.0288\ \text{kg}\cdot\text{m}^2$.

(ii) $I = \frac{1}{2}MR^2 = \frac{1}{2}(8.00\ \text{kg})(0.0600\ \text{m})^2 = 0.0144\ \text{kg}\cdot\text{m}^2$.

EVALUATE: *I* depends on how the mass of the object is distributed relative to the axis.

9.32. **IDENTIFY:** Treat each block as a point mass, so for each block $I = mr^2$, where *r* is the distance of the block from the axis. The total *I* for the object is the sum of the *I* for each of its pieces.
SET UP: In part (a) two blocks are a distance $L/2$ from the axis and the third block is on the axis. In part (b) two blocks are a distance $L/4$ from the axis and one is a distance $3L/4$ from the axis.

EXECUTE: **(a)** $I = 2m(L/2)^2 = \frac{1}{2}mL^2$.

(b) $I = 2m(L/4)^2 + m(3L/4)^2 = \dfrac{1}{16}mL^2(2+9) = \dfrac{11}{16}mL^2$.

EVALUATE: For the same object *I* is in general different for different axes.

9.33. **IDENTIFY:** *I* for the object is the sum of the values of *I* for each part.
SET UP: For the bar, for an axis perpendicular to the bar, use the appropriate expression from Table 9.2. For a point mass, $I = mr^2$, where *r* is the distance of the mass from the axis.

EXECUTE: **(a)** $I = I_{\text{bar}} + I_{\text{balls}} = \dfrac{1}{12}M_{\text{bar}}L^2 + 2m_{\text{balls}}\left(\dfrac{L}{2}\right)^2$.

$$I = \frac{1}{12}(4.00\ \text{kg})(2.00\ \text{m})^2 + 2(0.500\ \text{kg})(1.00\ \text{m})^2 = 2.33\ \text{kg}\cdot\text{m}^2$$

(b) $I = \dfrac{1}{3}m_{\text{bar}}L^2 + m_{\text{ball}}L^2 = \dfrac{1}{3}(4.00\ \text{kg})(2.00\ \text{m})^2 + (0.500\ \text{kg})(2.00\ \text{m})^2 = 7.33\ \text{kg}\cdot\text{m}^2$

(c) $I = 0$ because all masses are on the axis.

(d) All the mass is a distance $d = 0.500$ m from the axis and

$I = m_{\text{bar}}d^2 + 2m_{\text{ball}}d^2 = M_{\text{Total}}d^2 = (5.00\ \text{kg})(0.500\ \text{m})^2 = 1.25\ \text{kg}\cdot\text{m}^2$.

EVALUATE: *I* for an object depends on the location and direction of the axis.

9.34. **IDENTIFY:** Compare this object to a uniform disk of radius *R* and mass $2M$.
SET UP: With an axis perpendicular to the round face of the object at its center, *I* for a uniform disk is the same as for a solid cylinder.

EXECUTE: **(a)** The total *I* for a disk of mass $2M$ and radius *R*, $I = \frac{1}{2}(2M)R^2 = MR^2$. Each half of the disk has the same *I*, so for the half-disk, $I = \frac{1}{2}MR^2$.

(b) The same mass *M* is distributed the same way as a function of distance from the axis.

(c) The same method as in part (a) says that *I* for a quarter-disk of radius *R* and mass *M* is half that of a half-disk of radius *R* and mass $2M$, so $I = \frac{1}{2}(\frac{1}{2}[2M]R^2) = \frac{1}{2}MR^2$.

EVALUATE: *I* depends on how the mass of the object is distributed relative to the axis, and this is the same for any segment of a disk.

9.35. **IDENTIFY** and **SET UP:** $I = \sum m_i r_i^2$ implies $I = I_{\text{rim}} + I_{\text{spokes}}$

EXECUTE: $I_{\text{rim}} = MR^2 = (1.40\ \text{kg})(0.300\ \text{m})^2 = 0.126\ \text{kg}\cdot\text{m}^2$

Each spoke can be treated as a slender rod with the axis through one end, so

$I_{\text{spokes}} = 8\left(\frac{1}{3}ML^2\right) = \frac{8}{3}(0.280\ \text{kg})(0.300\ \text{m})^2 = 0.0672\ \text{kg}\cdot\text{m}^2$

$I = I_{\text{rim}} + I_{\text{spokes}} = 0.126\ \text{kg}\cdot\text{m}^2 + 0.0672\ \text{kg}\cdot\text{m}^2 = 0.193\ \text{kg}\cdot\text{m}^2$

EVALUATE: Our result is smaller than $m_{\text{tot}}R^2 = (3.64\text{ kg})(0.300\text{ m})^2 = 0.328\text{ kg}\cdot\text{m}^2$, since the mass of each spoke is distributed between $r = 0$ and $r = R$.

9.36. **IDENTIFY:** $K = \frac{1}{2}I\omega^2$. Use Table 9.2b to calculate I.

SET UP: $I = \frac{1}{12}ML^2$. 1 rpm = 0.1047 rad/s

EXECUTE: **(a)** $I = \frac{1}{12}(117\text{ kg})(2.08\text{ m})^2 = 42.2\text{ kg}\cdot\text{m}^2$. $\omega = (2400\text{ rev/min})\left(\dfrac{0.1047\text{ rad/s}}{1\text{ rev/min}}\right) = 251\text{ rad/s}$.

$K = \frac{1}{2}I\omega^2 = \frac{1}{2}(42.2\text{ kg}\cdot\text{m}^2)(251\text{ rad/s})^2 = 1.33\times10^6\text{ J}$.

(b) $K_1 = \frac{1}{12}M_1L_1^2\omega_1^2$, $K_2 = \frac{1}{12}M_2L_2^2\omega_2^2$. $L_1 = L_2$ and $K_1 = K_2$, so $M_1\omega_1^2 = M_2\omega_2^2$.

$$\omega_2 = \omega_1\sqrt{\frac{M_1}{M_2}} = (2400\text{ rpm})\sqrt{\frac{M_1}{0.750M_1}} = 2770\text{ rpm}$$

EVALUATE: The rotational kinetic energy is proportional to the square of the angular speed and directly proportional to the mass of the object.

9.37. **IDENTIFY:** I for the compound disk is the sum of I of the solid disk and of the ring.

SET UP: For the solid disk, $I = \frac{1}{2}m_\text{d}r_\text{d}^2$. For the ring, $I_\text{r} = \frac{1}{2}m_\text{r}(r_1^2 + r_2^2)$, where $r_1 = 50.0\text{ cm}, r_2 = 70.0\text{ cm}$. The mass of the disk and ring is their area times their area density.

EXECUTE: $I = I_\text{d} + I_\text{r}$.

Disk: $m_\text{d} = (3.00\text{ g/cm}^2)\pi r_\text{d}^2 = 23.56\text{ kg}$. $I_\text{d} = \dfrac{1}{2}m_\text{d}r_\text{d}^2 = 2.945\text{ kg}\cdot\text{m}^2$.

Ring: $m_\text{r} = (2.00\text{ g/cm}^2)\pi(r_2^2 - r_1^2) = 15.08\text{ kg}$. $I_\text{r} = \dfrac{1}{2}m_\text{r}(r_1^2 + r_2^2) = 5.580\text{ kg}\cdot\text{m}^2$.

$I = I_\text{d} + I_\text{r} = 8.52\text{ kg}\cdot\text{m}^2$.

EVALUATE: Even though $m_\text{r} < m_\text{d}$, $I_\text{r} > I_\text{d}$ since the mass of the ring is farther from the axis.

9.38. **IDENTIFY:** We can use angular kinematics (for constant angular acceleration) to find the angular velocity of the wheel. Then knowing its kinetic energy, we can find its moment of inertia, which is the target variable.

SET UP: $\theta - \theta_0 = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t$ and $K = \dfrac{1}{2}I\omega^2$.

EXECUTE: Converting the angle to radians gives $\theta - \theta_0 = (8.20\text{ rev})(2\pi\text{ rad/1 rev}) = 51.52\text{ rad}$.

$\theta - \theta_0 = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t$ gives $\omega_z = \dfrac{2(\theta - \theta_0)}{t} = \dfrac{2(51.52\text{ rad})}{12.0\text{ s}} = 8.587\text{ rad/s}$. Solving $K = \dfrac{1}{2}I\omega^2$ for I gives

$$I = \frac{2K}{\omega^2} = \frac{2(36.0\text{ J})}{(8.587\text{ rad/s})^2} = 0.976\text{ kg}\cdot\text{m}^2.$$

EVALUATE: The angular velocity must be in radians to use the formula $K = \dfrac{1}{2}I\omega^2$.

9.39. **IDENTIFY:** Knowing the kinetic energy, mass and radius of the sphere, we can find its angular velocity. From this we can find the tangential velocity (the target variable) of a point on the rim.

SET UP: $K = \frac{1}{2}I\omega^2$ and $I = \frac{2}{5}MR^2$ for a solid uniform sphere. The tagential velocity is $v = r\omega$.

EXECUTE: $I = \frac{2}{5}MR^2 = \frac{2}{5}(28.0\text{ kg})(0.380\text{ m})^2 = 1.617\text{ kg}\cdot\text{m}^2$. $K = \frac{1}{2}I\omega^2$ so

$$\omega = \sqrt{\frac{2K}{I}} = \sqrt{\frac{2(176\text{ J})}{1.617\text{ kg}\cdot\text{m}^2}} = 14.75\text{ rad/s}.$$

$v = r\omega = (0.380\text{ m})(14.75\text{ rad/s}) = 5.61\text{ m/s}$.

EVALUATE: This is the speed of a point on the surface of the sphere that is farthest from the axis of rotation (the "equator" of the sphere). Points off the "equator" would have smaller tangential velocity but the same angular velocity.

9.40. **IDENTIFY:** Knowing the angular acceleration of the sphere, we can use angular kinematics (with constant angular acceleration) to find its angular velocity. Then using its mass and radius, we can find its kinetic energy, the target variable.

SET UP: $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$, $K = \frac{1}{2}I\omega^2$, and $I = \frac{2}{3}MR^2$ for a uniform hollow spherical shell.

EXECUTE: $I = \frac{2}{3}MR^2 = \frac{2}{3}(8.20\text{ kg})(0.220\text{ m})^2 = 0.2646\text{ kg}\cdot\text{m}^2$. Converting the angle to radians gives $\theta - \theta_0 = (6.00\text{ rev})(2\pi\text{ rad/1 rev}) = 37.70\text{ rad}$. The angular velocity is $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$, which gives $\omega_z = \sqrt{2\alpha_z(\theta - \theta_0)} = \sqrt{2(0.890\text{ rad/s}^2)(37.70\text{ rad})} = 8.192\text{ rad/s}$.

$K = \frac{1}{2}(0.2646\text{ kg}\cdot\text{m}^2)(8.192\text{ rad/s})^2 = 8.88\text{ J}$.

EVALUATE: The angular velocity must be in radians to use the formula $K = \frac{1}{2}I\omega^2$.

9.41. **IDENTIFY:** $K = \frac{1}{2}I\omega^2$. Use Table 9.2 to calculate I.

SET UP: $I = \frac{2}{5}MR^2$. For the moon, $M = 7.35\times10^{22}\text{ kg}$ and $R = 1.74\times10^6\text{ m}$. The moon moves through $1\text{ rev} = 2\pi\text{ rad}$ in 27.3 d. $1\text{ d} = 8.64\times10^4\text{ s}$.

EXECUTE: **(a)** $I = \frac{2}{5}(7.35\times10^{22}\text{ kg})(1.74\times10^6\text{ m})^2 = 8.90\times10^{34}\text{ kg}\cdot\text{m}^2$.

$$\omega = \frac{2\pi\text{ rad}}{(27.3\text{ d})(8.64\times10^4\text{ s/d})} = 2.66\times10^{-6}\text{ rad/s}.$$

$K = \frac{1}{2}I\omega^2 = \frac{1}{2}(8.90\times10^{34}\text{ kg}\cdot\text{m}^2)(2.66\times10^{-6}\text{ rad/s})^2 = 3.15\times10^{23}\text{ J}$.

(b) $\dfrac{3.15\times10^{23}\text{ J}}{5(4.0\times10^{20}\text{ J})} = 158$ years. Considering the expense involved in tapping the moon's rotational energy, this does not seem like a worthwhile scheme for only 158 years worth of energy.

EVALUATE: The moon has a very large amount of kinetic energy due to its motion. The earth has even more, but changing the rotation rate of the earth would change the length of a day.

9.42. **IDENTIFY:** $K = \frac{1}{2}I\omega^2$. Use Table 9.2 to relate I to the mass M of the disk.

SET UP: $45.0\text{ rpm} = 4.71\text{ rad/s}$. For a uniform solid disk, $I = \frac{1}{2}MR^2$.

EXECUTE: **(a)** $I = \dfrac{2K}{\omega^2} = \dfrac{2(0.250\text{ J})}{(4.71\text{ rad/s})^2} = 0.0225\text{ kg}\cdot\text{m}^2$.

(b) $I = \frac{1}{2}MR^2$ and $M = \dfrac{2I}{R^2} = \dfrac{2(0.0225\text{ kg}\cdot\text{m}^2)}{(0.300\text{ m})^2} = 0.500\text{ kg}$.

EVALUATE: No matter what the shape is, the rotational kinetic energy is proportional to the mass of the object.

9.43. **IDENTIFY:** $K = \frac{1}{2}I\omega^2$, with ω in rad/s. Solve for I.

SET UP: $1\text{ rev/min} = (2\pi/60)\text{ rad/s}$. $\Delta K = -500\text{ J}$

EXECUTE: $\omega_i = 650\text{ rev/min} = 68.1\text{ rad/s}$. $\omega_f = 520\text{ rev/min} = 54.5\text{ rad/s}$. $\Delta K = K_f - K_i = \frac{1}{2}I(\omega_f^2 - \omega_i^2)$ and $I = \dfrac{2(\Delta K)}{\omega_f^2 - \omega_i^2} = \dfrac{2(-500\text{ J})}{(54.5\text{ rad/s})^2 - (68.1\text{ rad/s})^2} = 0.600\text{ kg}\cdot\text{m}^2$.

EVALUATE: In $K = \frac{1}{2}I\omega^2$, ω must be in rad/s.

9.44. **IDENTIFY:** The work done on the cylinder equals its gain in kinetic energy.

SET UP: The work done on the cylinder is PL, where L is the length of the rope. $K_1 = 0$. $K_2 = \frac{1}{2}I\omega^2$.

$I = mr^2 = \left(\dfrac{w}{g}\right)r^2$.

EXECUTE: $PL = \frac{1}{2}\frac{w}{g}v^2$, or $P = \frac{1}{2}\frac{w}{g}\frac{v^2}{L} = \frac{(40.0\text{ N})(6.00\text{ m/s})^2}{2(9.80\text{ m/s}^2)(5.00\text{ m})} = 14.7\text{ N}.$

EVALUATE: The linear speed v of the end of the rope equals the tangential speed of a point on the rim of the cylinder. When K is expressed in terms of v, the radius r of the cylinder doesn't appear.

9.45. **IDENTIFY** and **SET UP:** Combine Eqs. (9.17) and (9.15) to solve for K. Use Table 9.2 to get I.

EXECUTE: $K = \frac{1}{2}I\omega^2$

$a_{\text{rad}} = R\omega^2$, so $\omega = \sqrt{a_{\text{rad}}/R} = \sqrt{(3500\text{ m/s}^2)/1.20\text{ m}} = 54.0\text{ rad/s}$

For a disk, $I = \frac{1}{2}MR^2 = \frac{1}{2}(70.0\text{ kg})(1.20\text{ m})^2 = 50.4\text{ kg}\cdot\text{m}^2$

Thus $K = \frac{1}{2}I\omega^2 = \frac{1}{2}(50.4\text{ kg}\cdot\text{m}^2)(54.0\text{ rad/s})^2 = 7.35\times10^4\text{ J}$

EVALUATE: The limit on a_{rad} limits ω which in turn limits K.

9.46. **IDENTIFY:** Repeat the calculation in Example 9.8, but with a different expression for I.

SET UP: For the solid cylinder in Example 9.8, $I = \frac{1}{2}MR^2$. For the thin-walled, hollow cylinder, $I = MR^2$.

EXECUTE: **(a)** With $I = MR^2$, the expression for v is $v = \sqrt{\frac{2gh}{1+M/m}}$.

(b) This expression is smaller than that for the solid cylinder; more of the cylinder's mass is concentrated at its edge, so for a given speed, the kinetic energy of the cylinder is larger. A larger fraction of the potential energy is converted to the kinetic energy of the cylinder, and so less is available for the falling mass.

EVALUATE: When M is much larger than m, v is very small. When M is much less than m, v becomes $v = \sqrt{2gh}$, the same as for a mass that falls freely from a height h.

9.47. **IDENTIFY:** Apply conservation of energy to the system of stone plus pulley. $v = r\omega$ relates the motion of the stone to the rotation of the pulley.

SET UP: For a uniform solid disk, $I = \frac{1}{2}MR^2$. Let point 1 be when the stone is at its initial position and point 2 be when it has descended the desired distance. Let $+y$ be upward and take $y = 0$ at the initial position of the stone, so $y_1 = 0$ and $y_2 = -h$, where h is the distance the stone descends.

EXECUTE: **(a)** $K_p = \frac{1}{2}I_p\omega^2$. $I_p = \frac{1}{2}M_pR^2 = \frac{1}{2}(2.50\text{ kg})(0.200\text{ m})^2 = 0.0500\text{ kg}\cdot\text{m}^2$.

$\omega = \sqrt{\frac{2K_p}{I_p}} = \sqrt{\frac{2(4.50\text{ J})}{0.0500\text{ kg}\cdot\text{m}^2}} = 13.4\text{ rad/s}$. The stone has speed $v = R\omega = (0.200\text{ m})(13.4\text{ rad/s}) = 2.68\text{ m/s}$.

The stone has kinetic energy $K_s = \frac{1}{2}mv^2 = \frac{1}{2}(1.50\text{ kg})(2.68\text{ m/s})^2 = 5.39\text{ J}$. $K_1 + U_1 = K_2 + U_2$ gives

$0 = K_2 + U_2$. $0 = 4.50\text{ J} + 5.39\text{ J} + mg(-h)$. $h = \frac{9.89\text{ J}}{(1.50\text{ kg})(9.80\text{ m/s}^2)} = 0.673\text{ m}$.

(b) $K_{\text{tot}} = K_p + K_s = 9.89\text{ J}$. $\frac{K_p}{K_{\text{tot}}} = \frac{4.50\text{ J}}{9.89\text{ J}} = 45.5\%$.

EVALUATE: The gravitational potential energy of the pulley doesn't change as it rotates. The tension in the wire does positive work on the pulley and negative work of the same magnitude on the stone, so no net work on the system.

9.48. **IDENTIFY:** $K_p = \frac{1}{2}I\omega^2$ for the pulley and $K_b = \frac{1}{2}mv^2$ for the bucket. The speed of the bucket and the rotational speed of the pulley are related by $v = R\omega$.

SET UP: $K_p = \frac{1}{2}K_b$

EXECUTE: $\frac{1}{2}I\omega^2 = \frac{1}{2}(\frac{1}{2}mv^2) = \frac{1}{4}mR^2\omega^2$. $I = \frac{1}{2}mR^2$.

EVALUATE: The result is independent of the rotational speed of the pulley and the linear speed of the mass.

9.49. IDENTIFY: With constant acceleration, we can use kinematics to find the speed of the falling object. Then we can apply the work-energy expression to the entire system and find the moment of inertia of the wheel. Finally, using its radius we can find its mass, the target variable.

SET UP: With constant acceleration, $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t.$ The angular velocity of the wheel is related to the linear velocity of the falling mass by $\omega_z = \dfrac{v_y}{R}.$ The work-energy theorem is $K_1 + U_1 + W_{\text{other}} = K_2 + U_2,$ and the moment of inertia of a uniform disk is $I = \dfrac{1}{2}MR^2.$

EXECUTE: Find $v_y,$ the velocity of the block after it has descended 3.00 m. $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$ gives $v_y = \dfrac{2(y - y_0)}{t} = \dfrac{2(3.00\text{ m})}{2.00\text{ s}} = 3.00\text{ m/s}.$ For the wheel, $\omega_z = \dfrac{v_y}{R} = \dfrac{3.00\text{ m/s}}{0.280\text{ m}} = 10.71\text{ rad/s}.$ Apply the work-energy expression: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2,$ giving $mg(3.00\text{ m}) = \dfrac{1}{2}mv^2 + \dfrac{1}{2}I\omega^2.$ Solving for I gives $I = \dfrac{2}{\omega^2}\left[mg(3.00\text{ m}) - \dfrac{1}{2}mv^2\right].$

$I = \dfrac{2}{(10.71\text{ rad/s})^2}\left[(4.20\text{ kg})(9.8\text{ m/s}^2)(3.00\text{ m}) - \dfrac{1}{2}(4.20\text{ kg})(3.00\text{ m/s})^2\right].$ $I = 1.824\text{ kg}\cdot\text{m}^2.$ For a solid disk, $I = \frac{1}{2}MR^2$ gives $M = \dfrac{2I}{R^2} = \dfrac{2(1.824\text{ kg}\cdot\text{m}^2)}{(0.280\text{ m})^2} = 46.5\text{ kg}.$

EVALUATE: The gravitational potential of the falling object is converted into the kinetic energy of that object and the rotational kinetic energy of the wheel.

9.50. IDENTIFY: The work the person does is the negative of the work done by gravity. $W_{\text{grav}} = U_{\text{grav,1}} - U_{\text{grav,2}}.$ $U_{\text{grav}} = Mgy_{\text{cm}}.$

SET UP: The center of mass of the ladder is at its center, 1.00 m from each end. $y_{\text{cm,1}} = (1.00\text{ m})\sin 53.0° = 0.799\text{ m}.$ $y_{\text{cm,2}} = 1.00\text{ m}.$

EXECUTE: $W_{\text{grav}} = (9.00\text{ kg})(9.80\text{ m/s}^2)(0.799\text{ m} - 1.00\text{ m}) = -17.7\text{ J}.$ The work done by the person is 17.7 J.

EVALUATE: The gravity force is downward and the center of mass of the ladder moves upward, so gravity does negative work. The person pushes upward and does positive work.

9.51. IDENTIFY: The general expression for I is Eq. (9.16). $K = \frac{1}{2}I\omega^2.$

SET UP: R will be multiplied by f.

EXECUTE: **(a)** In the expression of Eq. (9.16), each term will have the mass multiplied by f^3 and the distance multiplied by f, and so the moment of inertia is multiplied by $f^3(f)^2 = f^5.$

(b) $(2.5\text{ J})(48)^5 = 6.37\times10^8$ J.

EVALUATE: Mass and volume are proportional to each other so both scale by the same factor.

9.52. IDENTIFY: $U = Mgy_{\text{cm}}.$ $\Delta U = U_2 - U_1.$

SET UP: Half the rope has mass 1.50 kg and length 12.0 m. Let $y = 0$ at the top of the cliff and take $+y$ to be upward. The center of mass of the hanging section of rope is at its center and $y_{\text{cm,2}} = -6.00$ m.

EXECUTE: $\Delta U = U_2 - U_1 = mg(y_{\text{cm,2}} - y_{\text{cm,1}}) = (1.50\text{ kg})(9.80\text{ m/s}^2)(-6.00\text{ m} - 0) = -88.2\text{ J}.$

EVALUATE: The potential energy of the rope decreases when part of the rope moves downward.

9.53. IDENTIFY: Use Eq. (9.19) to relate I for the wood sphere about the desired axis to I for an axis along a diameter.

SET UP: For a thin-walled hollow sphere, axis along a diameter, $I = \frac{2}{3}MR^2$.

For a solid sphere with mass M and radius R, $I_{cm} = \frac{2}{5}MR^2$, for an axis along a diameter.

EXECUTE: Find d such that $I_P = I_{cm} + Md^2$ with $I_P = \frac{2}{3}MR^2$:

$\frac{2}{3}MR^2 = \frac{2}{5}MR^2 + Md^2$

The factors of M divide out and the equation becomes $(\frac{2}{3} - \frac{2}{5})R^2 = d^2$

$d = \sqrt{(10-6)/15}R = 2R/\sqrt{15} = 0.516R$.

The axis is parallel to a diameter and is $0.516R$ from the center.

EVALUATE: $I_{cm}(\text{lead}) > I_{cm}(\text{wood})$ even though M and R are the same since for a hollow sphere all the mass is a distance R from the axis. Eq. (9.19) says $I_P > I_{cm}$, so there must be a d where $I_P(\text{wood}) = I_{cm}(\text{lead})$.

9.54. **IDENTIFY:** Apply Eq. (9.19), the parallel-axis theorem.

SET UP: The center of mass of the hoop is at its geometrical center.

EXECUTE: In Eq. (9.19), $I_{cm} = MR^2$ and $d = R^2$, so $I_P = 2MR^2$.

EVALUATE: I is larger for an axis at the edge than for an axis at the center. Some mass is closer than distance R from the axis but some is also farther away. Since I for each piece of the hoop is proportional to the square of the distance from the axis, the increase in distance has a larger effect.

9.55. **IDENTIFY** and **SET UP:** Use Eq. (9.19). The cm of the sheet is at its geometrical center. The object is sketched in Figure 9.55.

EXECUTE: $I_P = I_{cm} + Md^2$.

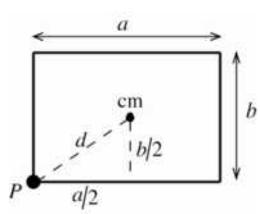


Figure 9.55

From part (c) of Table 9.2, $I_{cm} = \frac{1}{12}M(a^2 + b^2)$.

The distance d of P from the cm is $d = \sqrt{(a/2)^2 + (b/2)^2}$.

Thus $I_P = I_{cm} + Md^2 = \frac{1}{12}M(a^2+b^2) + M(\frac{1}{4}a^2 + \frac{1}{4}b^2) = (\frac{1}{12} + \frac{1}{4})M(a^2+b^2) = \frac{1}{3}M(a^2+b^2)$

EVALUATE: $I_P = 4I_{cm}$. For an axis through P mass is farther from the axis.

9.56. **IDENTIFY:** Consider the plate as made of slender rods placed side-by-side.

SET UP: The expression in Table 9.2(a) gives I for a rod and an axis through the center of the rod.

EXECUTE: **(a)** I is the same as for a rod with length a: $I = \frac{1}{12}Ma^2$.

(b) I is the same as for a rod with length b: $I = \frac{1}{12}Mb^2$.

EVALUATE: I is smaller when the axis is through the center of the plate than when it is along one edge.

9.57. **IDENTIFY:** Use the equations in Table 9.2. I for the rod is the sum of I for each segment. The parallel-axis theorem says $I_p = I_{cm} + Md^2$.

SET UP: The bent rod and axes a and b are shown in Figure 9.57. Each segment has length $L/2$ and mass $M/2$.

EXECUTE: **(a)** For each segment the moment of inertia is for a rod with mass $M/2$, length $L/2$ and the axis through one end. For one segment, $I_s = \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 = \frac{1}{24}ML^2$. For the rod, $I_a = 2I_s = \frac{1}{12}ML^2$.

(b) The center of mass of each segment is at the center of the segment, a distance of $L/4$ from each end. For each segment, $I_{\text{cm}} = \dfrac{1}{12}\left(\dfrac{M}{2}\right)\left(\dfrac{L}{2}\right)^2 = \dfrac{1}{96}ML^2$. Axis b is a distance $L/4$ from the cm of each segment, so for each segment the parallel axis theorem gives I for axis b to be $I_{\text{s}} = \dfrac{1}{96}ML^2 + \dfrac{M}{2}\left(\dfrac{L}{4}\right)^2 = \dfrac{1}{24}ML^2$ and $I_{\text{b}} = 2I_{\text{s}} = \dfrac{1}{12}ML^2$.

EVALUATE: I for these two axes are the same.

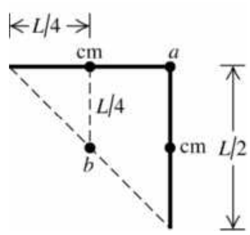


Figure 9.57

9.58. IDENTIFY: Eq. (9.20), $I = \int r^2 \, dm$

SET UP:

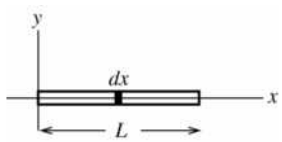


Figure 9.58

Take the x-axis to lie along the rod, with the origin at the left end. Consider a thin slice at coordinate x and width dx, as shown in Figure 9.58. The mass per unit length for this rod is M/L, so the mass of this slice is $dm = (M/L)\,dx$.

EXECUTE: $I = \int_0^L x^2 (M/L)\,dx = (M/L)\int_0^L x^2 \,dx = (M/L)(L^3/3) = \frac{1}{3}ML^2$

EVALUATE: This result agrees with Table 9.2.

9.59. IDENTIFY: Apply Eq. (9.20).

SET UP: $dm = \rho dV = \rho(2\pi rL\,dr)$, where L is the thickness of the disk. $M = \pi L\rho R^2$.

EXECUTE: The analysis is identical to that of Example 9.10, with the lower limit in the integral being zero and the upper limit being R. The result is $I = \frac{1}{2}MR^2$.

EVALUATE: Our result agrees with Table 9.2(f).

9.60. IDENTIFY: Apply Eq. (9.20).

SET UP: For this case, $dm = \gamma\,dx$.

EXECUTE: **(a)** $M = \int dm = \int_0^L \gamma x\,dx = \gamma \left.\dfrac{x^2}{2}\right|_0^L = \dfrac{\gamma L^2}{2}$

(b) $I = \int_0^L x^2(\gamma x)dx = \gamma \left.\dfrac{x^4}{4}\right|_0^L = \dfrac{\gamma L^4}{4} = \dfrac{M}{2}L^2$. This is larger than the moment of inertia of a uniform rod of the same mass and length, since the mass density is greater farther away from the axis than nearer the axis.

(c) $I=\int_0^L (L-x)^2\gamma x dx=\gamma\int_0^L (L^2x-2Lx^2+x^3)dx=\gamma\left(L^2\frac{x^2}{2}-2L\frac{x^3}{3}+\frac{x^4}{4}\right)\Bigg|_0^L=\gamma\frac{L^4}{12}=\frac{M}{6}L^2$.

This is a third of the result of part (b), reflecting the fact that more of the mass is concentrated at the right end.

EVALUATE: For a uniform rod with an axis at one end, $I=\frac{1}{3}ML^2$. The result in (b) is larger than this and the result in (c) is smaller than this.

9.61. **IDENTIFY:** We know the angular acceleration as a function of time and want to find the angular velocity and the angle the flywheel has turned through at a later time.

SET UP: $\omega_z(t)=\omega_{0z}+\int_0^t \alpha_z(t')dt'$ and $\theta-\theta_0=\int_0^t \omega_z(t')dt'$.

EXECUTE: **(a)** Integrating the angular acceleration gives the angular velocity:

$\omega_z(t)=\omega_{0z}+\int_0^t \alpha_z(t')dt'=\int_0^t [(8.60\text{ rad/s}^2)-(2.30\text{ rad/s}^3)t']dt'=(8.60\text{ rad/s}^2)t-(1.15\text{ rad/s}^3)t^2$

At $t=5.00$ s, $\omega_z=(8.60\text{ rad/s}^2)(5.00\text{ s})-(1.15\text{ rad/s}^3)(5.00\text{ s})^2=14.2\text{ rad/s}$.

(b) Integrating the angular velocity gives the angle:

$\theta-\theta_0=\int_0^t \omega_z(t')dt'=(4.30\text{ rad/s}^2)t^2-(0.3833\text{ rad/s}^3)t^3$. At $t=5.00$ s,

$\theta-\theta_0=107.5\text{ rad}-47.9\text{ rad}=59.6\text{ rad}$.

EVALUATE: With non-constant angular acceleration, we cannot use the standard angular kinematics formulas, but must use integration instead.

9.62. **IDENTIFY:** Using the equation for the angle as a function of time, we can find the angular acceleration of the disk at a given time and use this to find the linear acceleration of a point on the rim (the target variable).

SET UP: We can use the definitions of the angular velocity and the angular acceleration: $\omega_z(t)=\frac{d\theta}{dt}$ and $\alpha_z(t)=\frac{d\omega_z}{dt}$. The acceleration components are $a_{\text{rad}}=R\omega^2$ and $a_{\tan}=R\alpha$, and the magnitude of the acceleration is $a=\sqrt{a_{\text{rad}}^2+a_{\tan}^2}$.

SET UP: $\omega_z(t)=\frac{d\theta}{dt}=1.10\text{ rad/s}+(17.2\text{ rad/s}^2)t$. $\alpha_z(t)=\frac{d\omega_z}{dt}=17.2\text{ rad/s}^2$ (constant).

$\theta=0.100\text{ rev}=0.6283\text{ rad}$ gives $8.60t^2+1.10t-0.6283=0$, so $t=-0.064\pm0.2778$ s. Since t must be positive, $t=0.2138$ s. At this t, $\omega_z(t)=4.777$ rad/s and $\alpha_z(t)=17.2\text{ rad/s}^2$. For a point on the rim,

$a_{\text{rad}}=R\omega^2=9.129\text{ m/s}^2$ and $a_{\tan}=R\alpha=6.88\text{ m/s}^2$, so $a=\sqrt{a_{\text{rad}}^2+a_{\tan}^2}=11.4\text{ m/s}^2$.

EVALUATE: Since the angular acceleration is constant, we could use the constant acceleration formulas as a check. For example, $\frac{1}{2}\alpha_z=8.60\text{ rad/s}^2$ gives $\alpha_z=17.2\text{ rad/s}^2$.

9.63. **IDENTIFY:** The target variable is the horizontal distance the piece travels before hitting the floor. Using the angular acceleration of the blade, we can find its angular velocity when the piece breaks off. This will give us the linear horizontal speed of the piece. It is then in free fall, so we can use the linear kinematics equations.

SET UP: $\omega_z^2=\omega_{0z}^2+2\alpha_z(\theta-\theta_0)$ for the blade, and $v=r\omega$ is the horizontal velocity of the piece.

$y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ for the falling piece.

EXECUTE: Find the initial horizontal velocity of the piece just after it breaks off.

$\theta-\theta_0=(155\text{ rev})(2\pi\text{ rad/1 rev})=973.9\text{ rad}$.

$\alpha_z=(3.00\text{ rev/s}^2)(2\pi\text{ rad/1 rev})=18.85\text{ rad/s}^2$. $\omega_z^2=\omega_{0z}^2+2\alpha_z(\theta-\theta_0)$.

$\omega_z = \sqrt{2\alpha_z(\theta - \theta_0)} = \sqrt{2(18.85\text{ rad/s}^2)(973.9\text{ rad})} = 191.6\text{ rad/s}$. The horizontal velocity of the piece is $v = r\omega = (0.120\text{ m})(191.6\text{ rad/s}) = 23.0\text{ m/s}$. Now consider the projectile motion of the piece. Take $+y$ downward and use the vertical motion to find t. Solving $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ for t gives

$t = \sqrt{\frac{2(y-y_0)}{a_y}} = \sqrt{\frac{2(0.820\text{ m})}{9.8\text{ m/s}^2}} = 0.4091\text{ s}$. Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (23.0\text{ m/s})(0.4091\text{ s}) = 9.41\text{ m}$.

EVALUATE: Once the piece is free of the blade, the only force acting on it is gravity so its acceleration is g downward.

9.64. **IDENTIFY** and **SET UP:** Use Eqs. (9.3) and (9.5). As long as $\alpha_z > 0$, ω_z increases. At the t when $\alpha_z = 0$, ω_z is at its maximum positive value and then starts to decrease when α_z becomes negative.

$\theta(t) = \gamma t^2 - \beta t^3$; $\gamma = 3.20\text{ rad/s}^2$, $\beta = 0.500\text{ rad/s}^3$

EXECUTE: **(a)** $\omega_z(t) = \frac{d\theta}{dt} = \frac{d(\gamma t^2 - \beta t^3)}{dt} = 2\gamma t - 3\beta t^2$

(b) $\alpha_z(t) = \frac{d\omega_z}{dt} = \frac{d(2\gamma t - 3\beta t^2)}{dt} = 2\gamma - 6\beta t$

(c) The maximum angular velocity occurs when $\alpha_z = 0$.

$2\gamma - 6\beta t = 0$ implies $t = \frac{2\gamma}{6\beta} = \frac{\gamma}{3\beta} = \frac{3.20\text{ rad/s}^2}{3(0.500\text{ rad/s}^3)} = 2.133\text{ s}$

At this t, $\omega_z = 2\gamma t - 3\beta t^2 = 2(3.20\text{ rad/s}^2)(2.133\text{ s}) - 3(0.500\text{ rad/s}^3)(2.133\text{ s})^2 = 6.83\text{ rad/s}$

The maximum positive angular velocity is 6.83 rad/s and it occurs at 2.13 s.

EVALUATE: For large t both ω_z and α_z are negative and ω_z increases in magnitude. In fact, $\omega_z \to -\infty$ at $t \to \infty$. So the answer in (c) is not the largest angular speed, just the largest positive angular velocity.

9.65. **IDENTIFY:** The angular acceleration α of the disk is related to the linear acceleration a of the ball by $a = R\alpha$. Since the acceleration is not constant, use $\omega_z - \omega_{0z} = \int_0^t \alpha_z dt$ and $\theta - \theta_0 = \int_0^t \omega_z dt$ to relate θ, ω_z, α_z and t for the disk. $\omega_{0z} = 0$.

SET UP: $\int t^n dt = \frac{1}{n+1}t^{n+1}$. In $a = R\alpha$, α is in rad/s^2.

EXECUTE: **(a)** $A = \frac{a}{t} = \frac{1.80\text{ m/s}^2}{3.00\text{ s}} = 0.600\text{ m/s}^3$

(b) $\alpha = \frac{a}{R} = \frac{(0.600\text{ m/s}^3)t}{0.250\text{ m}} = (2.40\text{ rad/s}^3)t$

(c) $\omega_z = \int_0^t (2.40\text{ rad/s}^3)t\,dt = (1.20\text{ rad/s}^3)t^2$. $\omega_z = 15.0\text{ rad/s}$ for $t = \sqrt{\frac{15.0\text{ rad/s}}{1.20\text{ rad/s}^3}} = 3.54\text{ s}$.

(d) $\theta - \theta_0 = \int_0^t \omega_z dt = \int_0^t (1.20\text{ rad/s}^3)t^2 dt = (0.400\text{ rad/s}^3)t^3$. For $t = 3.54\text{ s}$, $\theta - \theta_0 = 17.7\text{ rad}$.

EVALUATE: If the disk had turned at a constant angular velocity of 15.0 rad/s for 3.54 s it would have turned through an angle of 53.1 rad in 3.54 s. It actually turns through less than half this because the angular velocity is increasing in time and is less than 15.0 rad/s at all but the end of the interval.

9.66. **IDENTIFY** and **SET UP:** The translational kinetic energy is $K = \frac{1}{2}mv^2$ and the kinetic energy of the rotating flywheel is $K = \frac{1}{2}I\omega^2$. Use the scale speed to calculate the actual speed v. From that calculate K for the car and then solve for ω that gives this K for the flywheel.

EXECUTE: **(a)** $\frac{v_{\text{toy}}}{v_{\text{scale}}} = \frac{L_{\text{toy}}}{L_{\text{real}}}$

$$v_{\text{toy}} = v_{\text{scale}}\left(\frac{L_{\text{toy}}}{L_{\text{real}}}\right) = (700\text{ km/h})\left(\frac{0.150\text{ m}}{3.0\text{ m}}\right) = 35.0\text{ km/h}$$

$$v_{\text{toy}} = (35.0\text{ km/h})(1000\text{ m/1 km})(1\text{ h/3600 s}) = 9.72\text{ m/s}$$

(b) $K = \frac{1}{2}mv^2 = \frac{1}{2}(0.180\text{ kg})(9.72\text{ m/s})^2 = 8.50\text{ J}$

(c) $K = \frac{1}{2}I\omega^2$ gives that $\omega = \sqrt{\frac{2K}{I}} = \sqrt{\frac{2(8.50\text{ J})}{4.00\times 10^{-5}\text{ kg}\cdot\text{m}^2}} = 652\text{ rad/s}$

EVALUATE: $K = \frac{1}{2}I\omega^2$ gives ω in rad/s. $652\text{ rad/s} = 6200\text{ rev/min}$ so the rotation rate of the flywheel is very large.

9.67. **IDENTIFY:** $a_{\text{tan}} = r\alpha$, $a_{\text{rad}} = r\omega^2$. Apply the constant acceleration equations and $\sum\vec{F} = m\vec{a}$.

SET UP: a_{tan} and a_{rad} are perpendicular components of $\vec{a}$, so $a = \sqrt{a_{\text{rad}}^2 + a_{\text{tan}}^2}$.

EXECUTE: **(a)** $\alpha = \frac{a_{\text{tan}}}{r} = \frac{2.00\text{ m/s}^2}{60.0\text{ m}} = 0.0333\text{ rad/s}^2$.

(b) $\alpha t = (0.0333\text{ rad/s}^2)(6.00\text{ s}) = 0.200\text{ rad/s}$.

(c) $a_{\text{rad}} = \omega^2 r = (0.200\text{ rad/s})^2(60.0\text{ m}) = 2.40\text{ m/s}^2$.

(d) The sketch is given in Figure 9.67.

(e) $a = \sqrt{a_{\text{rad}}^2 + a_{\text{tan}}^2} = \sqrt{(2.40\text{ m/s}^2)^2 + (2.00\text{ m/s}^2)^2} = 3.12\text{ m/s}^2$, and the magnitude of the force is $F = ma = (1240\text{ kg})(3.12\text{ m/s}^2) = 3.87\text{ kN}$.

(f) $\arctan\left(\frac{a_{\text{rad}}}{a_{\text{tan}}}\right) = \arctan\left(\frac{2.40}{2.00}\right) = 50.2°$.

EVALUATE: a_{tan} is constant and a_{rad} increases as ω increases. At $t = 0$, $\vec{a}$ is parallel to $\vec{v}$. As t increases, $\vec{a}$ moves toward the radial direction and the angle between $\vec{a}$ and $\vec{v}$ increases toward 90°.

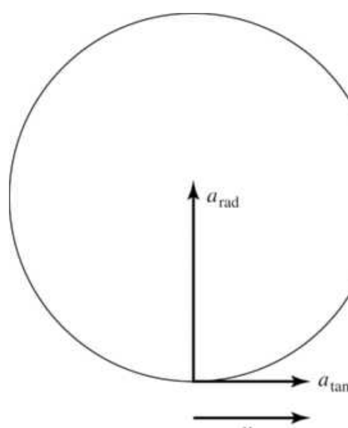


Figure 9.67

9.68. **IDENTIFY:** Apply conservation of energy to the system of drum plus falling mass, and compare the results for earth and for Mars.

SET UP: $K_{\text{drum}} = \frac{1}{2}I\omega^2$. $K_{\text{mass}} = \frac{1}{2}mv^2$. $v = R\omega$ so if K_{drum} is the same, ω is the same and v is the same on both planets. Therefore, K_{mass} is the same. Let $y = 0$ at the initial height of the mass and take $+y$ upward. Configuration 1 is when the mass is at its initial position and 2 is when the mass has descended 5.00 m, so $y_1 = 0$ and $y_2 = -h$, where h is the height the mass descends.

EXECUTE: **(a)** $K_1 + U_1 = K_2 + U_2$ gives $0 = K_{\text{drum}} + K_{\text{mass}} - mgh$. $K_{\text{drum}} + K_{\text{mass}}$ are the same on both planets, so $mg_{\text{E}}h_{\text{E}} = mg_{\text{M}}h_{\text{M}}$. $h_{\text{M}} = h_{\text{E}}\left(\dfrac{g_{\text{E}}}{g_{\text{M}}}\right) = (5.00\text{ m})\left(\dfrac{9.80\text{ m/s}^2}{3.71\text{ m/s}^2}\right) = 13.2\text{ m}$.

(b) $mg_{\text{M}}h_{\text{M}} = K_{\text{drum}} + K_{\text{mass}}$. $\frac{1}{2}mv^2 = mg_{\text{M}}h_{\text{M}} - K_{\text{drum}}$ and

$$v = \sqrt{2g_{\text{M}}h_{\text{M}} - \frac{2K_{\text{drum}}}{m}} = \sqrt{2(3.71\text{ m/s}^2)(13.2\text{ m}) - \frac{2(250.0\text{ J})}{15.0\text{ kg}}} = 8.04\text{ m/s}$$

EVALUATE: We did the calculations without knowing the moment of inertia I of the drum, or the mass and radius of the drum.

9.69. **IDENTIFY** and **SET UP:** All points on the belt move with the same speed. Since the belt doesn't slip, the speed of the belt is the same as the speed of a point on the rim of the shaft and on the rim of the wheel, and these speeds are related to the angular speed of each circular object by $v = r\omega$.

EXECUTE:

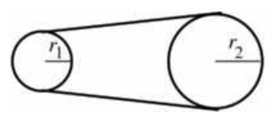


Figure 9.69

(a) $v_1 = r_1\omega_1$

$\omega_1 = (60.0\text{ rev/s})(2\pi\text{ rad/1 rev}) = 377\text{ rad/s}$

$v_1 = r_1\omega_1 = (0.45\times 10^{-2}\text{ m})(377\text{ rad/s}) = 1.70\text{ m/s}$

(b) $v_1 = v_2$

$r_1\omega_1 = r_2\omega_2$

$\omega_2 = (r_1/r_2)\omega_1 = (0.45\text{ cm}/1.80\text{ cm})(377\text{ rad/s}) = 94.2\text{ rad/s}$

EVALUATE: The wheel has a larger radius than the shaft so turns slower to have the same tangential speed for points on the rim.

9.70. **IDENTIFY:** The speed of all points on the belt is the same, so $r_1\omega_1 = r_2\omega_2$ applies to the two pulleys.

SET UP: The second pulley, with half the diameter of the first, must have twice the angular velocity, and this is the angular velocity of the saw blade. π rad/s = 30 rev/min.

EXECUTE: **(a)** $v_2 = (2(3450\text{ rev/min}))\left(\dfrac{\pi}{30}\dfrac{\text{rad/s}}{\text{rev/min}}\right)\left(\dfrac{0.208\text{ m}}{2}\right) = 75.1\text{ m/s}$.

(b) $a_{\text{rad}} = \omega^2 r = \left(2(3450\text{ rev/min})\left(\dfrac{\pi}{30}\dfrac{\text{rad/s}}{\text{rev/min}}\right)\right)^2\left(\dfrac{0.208\text{ m}}{2}\right) = 5.43\times 10^4\text{ m/s}^2$,

so the force holding sawdust on the blade would have to be about 5500 times as strong as gravity.

EVALUATE: In $v = r\omega$ and $a_{\text{rad}} = r\omega^2$, ω must be in rad/s.

9.71. **IDENTIFY:** Apply $v = r\omega$.

SET UP: Points on the chain all move at the same speed, so $r_{\text{r}}\omega_{\text{r}} = r_{\text{f}}\omega_{\text{f}}$.

EXECUTE: The angular velocity of the rear wheel is $\omega_{\text{r}} = \dfrac{v_{\text{r}}}{r} = \dfrac{5.00\text{ m/s}}{0.330\text{ m}} = 15.15\text{ rad/s}$.

The angular velocity of the front wheel is $\omega_{\text{f}} = 0.600\text{ rev/s} = 3.77\text{ rad/s}$. $r_{\text{r}} = r_{\text{f}}(\omega_{\text{f}}/\omega_{\text{r}}) = 2.99\text{ cm}$.

EVALUATE: The rear sprocket and wheel have the same angular velocity and the front sprocket and wheel have the same angular velocity. $r\omega$ is the same for both, so the rear sprocket has a smaller radius since it has a larger angular velocity. The speed of a point on the chain is $v = r_{\text{r}}\omega_{\text{r}} = (2.99\times 10^{-2}\text{ m})(15.15\text{ rad/s}) =$ 0.453 m/s. The linear speed of the bicycle is 5.00 m/s.

9.72. **IDENTIFY:** Use the constant angular acceleration equations, applied to the first revolution and to the first two revolutions.

SET UP: Let the direction the disk is rotating be positive. $1\text{ rev} = 2\pi\text{ rad}$. Let t be the time for the first revolution. The time for the first two revolutions is $t + 0.750\text{ s}$.

EXECUTE: **(a)** $\theta - \theta_0 = \omega_{0z}t + \tfrac{1}{2}\alpha_z t^2$ applied to the first revolution and then to the first two revolutions gives $2\pi\text{ rad} = \tfrac{1}{2}\alpha_z t^2$ and $4\pi\text{ rad} = \tfrac{1}{2}\alpha_z (t + 0.750\text{ s})^2$. Eliminating α_z between these equations gives $4\pi\text{ rad} = \dfrac{2\pi\text{ rad}}{t^2}(t + 0.750\text{ s})^2$. $2t^2 = (t + 0.750\text{ s})^2$. $\sqrt{2}t = \pm(t + 0.750\text{ s})$. The positive root is $t = \dfrac{0.750\text{ s}}{\sqrt{2} - 1} = 1.81\text{ s}$.

(b) $2\pi\text{ rad} = \tfrac{1}{2}\alpha_z t^2$ and $t = 1.81\text{ s}$ gives $\alpha_z = 3.84\text{ rad/s}^2$

EVALUATE: At the start of the second revolution, $\omega_{0z} = (3.84\text{ rad/s}^2)(1.81\text{ s}) = 6.95\text{ rad/s}$. The distance the disk rotates in the next 0.750 s is $\theta - \theta_0 = \omega_{0z}t + \tfrac{1}{2}\alpha_z t^2 = (6.95\text{ rad/s})(0.750\text{ s}) + \tfrac{1}{2}(3.84\text{ rad/s}^2)(0.750\text{ s})^2 = 6.29\text{ rad}$, which is two revolutions.

9.73. **IDENTIFY** and **SET UP:** Use Eq. (9.15) to relate a_{rad} to ω and then use a constant acceleration equation to replace ω.

EXECUTE: **(a)** $a_{\text{rad}} = r\omega^2$, $a_{\text{rad},1} = r\omega_1^2$, $a_{\text{rad},2} = r\omega_2^2$. $\Delta a_{\text{rad}} = a_{\text{rad},2} - a_{\text{rad},1} = r(\omega_2^2 - \omega_1^2)$. One of the constant acceleration equations can be written $\omega_{2z}^2 = \omega_{1z}^2 + 2\alpha(\theta_2 - \theta_1)$, or $\omega_{2z}^2 - \omega_{1z}^2 = 2\alpha_z(\theta_2 - \theta_1)$. Thus $\Delta a_{\text{rad}} = r2\alpha_z(\theta_2 - \theta_1) = 2r\alpha_z(\theta_2 - \theta_1)$, as was to be shown.

(b) $\alpha_z = \dfrac{\Delta a_{\text{rad}}}{2r(\theta_2 - \theta_1)} = \dfrac{85.0\text{ m/s}^2 - 25.0\text{ m/s}^2}{2(0.250\text{ m})(20.0\text{ rad})} = 6.00\text{ rad/s}^2$. Then $a_{\tan} = r\alpha = (0.250\text{ m})(6.00\text{ rad/s}^2) = 1.50\text{ m/s}^2$.

EVALUATE: ω^2 is proportional to α_z and $(\theta - \theta_0)$ so a_{rad} is also proportional to these quantities. a_{rad} increases while r stays fixed, ω_z increases, and α_z is positive.

IDENTIFY and **SET UP:** Use Eq. (9.17) to relate K and ω and then use a constant acceleration equation to replace ω.

EXECUTE: **(c)** $K = \tfrac{1}{2}I\omega^2$; $K_2 = \tfrac{1}{2}I\omega_2^2$, $K_1 = \tfrac{1}{2}I\omega_1^2$

$\Delta K = K_2 - K_1 = \tfrac{1}{2}I(\omega_2^2 - \omega_1^2) = \tfrac{1}{2}I(2\alpha_z(\theta_2 - \theta_1)) = I\alpha_z(\theta_2 - \theta_1)$, as was to be shown.

(d) $I = \dfrac{\Delta K}{\alpha_z(\theta_2 - \theta_1)} = \dfrac{45.0\text{ J} - 20.0\text{ J}}{(6.00\text{ rad/s}^2)(20.0\text{ rad})} = 0.208\text{ kg}\cdot\text{m}^2$.

EVALUATE: α_z is positive, ω increases and K increases.

9.74. **IDENTIFY:** $I = I_{\text{wood}} + I_{\text{lead}}$. $m = \rho V$, where ρ is the volume density and $m = \sigma A$, where σ is the area density.

SET UP: For a solid sphere, $I = \tfrac{2}{5}mR^2$. For the hollow sphere (foil), $I = \tfrac{2}{3}mR^2$. For a sphere, $V = \tfrac{4}{3}\pi R^3$ and $A = 4\pi R^2$. $m_{\text{w}} = \rho_{\text{w}}V_{\text{w}} = \rho_{\text{w}}\dfrac{4}{3}\pi R^3$. $m_{\text{L}} = \sigma_{\text{L}}A_{\text{L}} = \sigma_{\text{L}}4\pi R^2$.

EXECUTE: $I = \dfrac{2}{5}m_{\text{w}}R^2 + \dfrac{2}{3}m_{\text{L}}R^2 = \dfrac{2}{5}\left(\rho_{\text{w}}\dfrac{4}{3}\pi R^3\right)R^2 + \dfrac{2}{3}(\sigma_{\text{L}}4\pi R^2)R^2 = \dfrac{8}{3}\pi R^4\left(\dfrac{\rho_{\text{w}}R}{5} + \sigma_{\text{L}}\right)$.

$$I = \frac{8\pi}{3}(0.30\text{ m})^4\left[\frac{(800\text{ kg/m}^3)(0.30\text{ m})}{5} + 20\text{ kg/m}^2\right] = 4.61\text{ kg}\cdot\text{m}^2.$$

EVALUATE: $m_{\text{W}} = 90.5\text{ kg}$ and $I_{\text{W}} = 3.26\text{ kg}\cdot\text{m}^2$. $m_{\text{L}} = 22.6\text{ kg}$ and $I_{\text{L}} = 1.36\text{ kg}\cdot\text{m}^2$. Even though the foil is only 20% of the total mass, its contribution to I is about 30% of the total.

9.75. **IDENTIFY:** $K = \frac{1}{2}I\omega^2$. $a_{\text{rad}} = r\omega^2$. $m = \rho V$.

SET UP: For a disk with the axis at the center, $I = \frac{1}{2}mR^2$. $V = t\pi R^2$, where $t = 0.100$ m is the thickness of the flywheel. $\rho = 7800\ \text{kg/m}^3$ is the density of the iron.

EXECUTE: **(a)** $\omega = 90.0\ \text{rpm} = 9.425\ \text{rad/s}$. $I = \dfrac{2K}{\omega^2} = \dfrac{2(10.0\times10^6\ \text{J})}{(9.425\ \text{rad/s})^2} = 2.252\times10^5\ \text{kg}\cdot\text{m}^2$.

$m = \rho V = \rho\pi R^2 t$. $I = \dfrac{1}{2}mR^2 = \dfrac{1}{2}\rho\pi t R^4$. This gives $R = (2I/\rho\pi t)^{1/4} = 3.68\ \text{m}$ and the diameter is 7.36 m.

(b) $a_{\text{rad}} = R\omega^2 = 327\ \text{m/s}^2$

EVALUATE: In $K = \frac{1}{2}I\omega^2$, ω must be in rad/s. a_{rad} is about 33g; the flywheel material must have large cohesive strength to prevent the flywheel from flying apart.

9.76. **IDENTIFY:** $K = \frac{1}{2}I\omega^2$. To have the same K for any ω the two parts must have the same I. Use Table 9.2 for I.

SET UP: For a solid sphere, $I_{\text{solid}} = \frac{2}{5}M_{\text{solid}}R^2$. For a hollow sphere, $I_{\text{hollow}} = \frac{2}{3}M_{\text{hollow}}R^2$.

EXECUTE: $I_{\text{solid}} = I_{\text{hollow}}$ gives $\frac{2}{5}M_{\text{solid}}R^2 = \frac{2}{3}M_{\text{hollow}}R^2$ and $M_{\text{hollow}} = \frac{3}{5}M_{\text{solid}} = \frac{3}{5}M$.

EVALUATE: The hollow sphere has less mass since all its mass is distributed farther from the rotation axis.

9.77. **IDENTIFY:** $K = \frac{1}{2}I\omega^2$. $\omega = \dfrac{2\pi\ \text{rad}}{T}$, where T is the period of the motion. For the earth's orbital motion it can be treated as a point mass and $I = MR^2$.

SET UP: The earth's rotational period is $24\ \text{h} = 86{,}164\ \text{s}$. Its orbital period is $1\ \text{yr} = 3.156\times10^7\ \text{s}$. $M = 5.97\times10^{24}\ \text{kg}$. $R = 6.38\times10^6\ \text{m}$.

EXECUTE: **(a)** $K = \dfrac{2\pi^2 I}{T^2} = \dfrac{2\pi^2(0.3308)(5.97\times10^{24}\ \text{kg})(6.38\times10^6\text{m})^2}{(86.164\ \text{s})^2} = 2.14\times10^{29}\ \text{J}$.

(b) $K = \dfrac{1}{2}M\left(\dfrac{2\pi R}{T}\right)^2 = \dfrac{2\pi^2(5.97\times10^{24}\ \text{kg})(1.50\times10^{11}\ \text{m})^2}{(3.156\times10^7\ \text{s})^2} = 2.66\times10^{33}\ \text{J}$.

(c) Since the earth's moment of inertia is less than that of a uniform sphere, more of the earth's mass must be concentrated near its center.

EVALUATE: These kinetic energies are very large, because the mass of the earth is very large.

9.78. **IDENTIFY:** Using energy considerations, the system gains as kinetic energy the lost potential energy, mgR.

SET UP: The kinetic energy is $K = \dfrac{1}{2}I\omega^2 + \dfrac{1}{2}mv^2$, with $I = \frac{1}{2}mR^2$ for the disk. $v = R\omega$.

EXECUTE: $K = \dfrac{1}{2}I\omega^2 + \dfrac{1}{2}m(\omega R)^2 = \dfrac{1}{2}(I + mR^2)\omega^2$. Using $I = \frac{1}{2}mR^2$ and solving for ω, $\omega^2 = \dfrac{4}{3}\dfrac{g}{R}$ and $\omega = \sqrt{\dfrac{4}{3}\dfrac{g}{R}}$.

EVALUATE: The small object has speed $v = \sqrt{\dfrac{2}{3}}\sqrt{2gR}$. If it was not attached to the disk and was dropped from a height h, it would attain a speed $\sqrt{2gR}$. Being attached to the disk reduces its final speed by a factor of $\sqrt{\dfrac{2}{3}}$.

9.79. **IDENTIFY:** Use Eq. (9.20) to calculate I. Then use $K = \frac{1}{2}I\omega^2$ to calculate K.

(a) SET UP: The object is sketched in Figure 9.79.

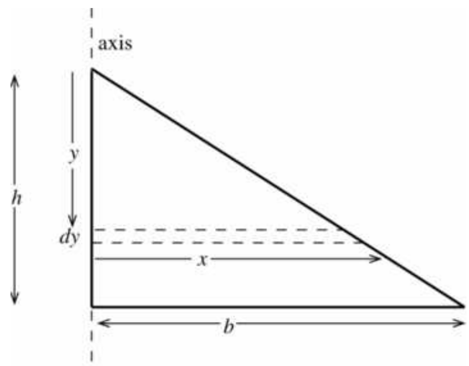


Figure 9.79

Consider a small strip of width dy and a distance y below the top of the triangle. The length of the strip is $x = (y/h)b$.

EXECUTE: The strip has area $x\,dy$ and the area of the sign is $\frac{1}{2}bh$, so the mass of the strip is

$$dm = M\left(\frac{x\,dy}{\frac{1}{2}bh}\right) = M\left(\frac{yb}{h}\right)\left(\frac{2\,dy}{bh}\right) = \left(\frac{2M}{h^2}\right)y\,dy$$

$$dI = \tfrac{1}{3}(dm)x^2 = \left(\frac{2Mb^2}{3h^4}\right)y^3\,dy$$

$$I = \int_0^h dI = \frac{2Mb^2}{3h^4}\int_0^h y^3\,dy = \frac{2Mb^2}{3h^4}\left(\frac{1}{4}y^4\Big|_0^h\right) = \frac{1}{6}Mb^2$$

(b) $I = \frac{1}{6}Mb^2 = 2.304\ \text{kg}\cdot\text{m}^2$

$\omega = 2.00\ \text{rev/s} = 4.00\pi\ \text{rad/s}$

$K = \frac{1}{2}I\omega^2 = 182\ \text{J}$

EVALUATE: From Table (9.2), if the sign were rectangular, with length b, then $I = \frac{1}{3}Mb^2$. Our result is one-half this, since mass is closer to the axis for the triangular than for the rectangular shape.

9.80. **IDENTIFY:** Apply conservation of energy to the system.

SET UP: For the falling mass $K = \frac{1}{2}mv^2$. For the wheel $K = \frac{1}{2}I\omega^2$.

EXECUTE: **(a)** The kinetic energy of the falling mass after 2.00 m is $K = \frac{1}{2}mv^2 = \frac{1}{2}(8.00\ \text{kg})(5.00\ \text{m/s})^2 =$ 100 J. The change in its potential energy while falling is $mgh = (8.00\ \text{kg})(9.8\ \text{m/s}^2)(2.00\ \text{m}) = 156.8\ \text{J}$. The wheel must have the "missing" 56.8 J in the form of rotational kinetic energy. Since its outer rim is moving at the same speed as the falling mass, 5.00 m/s, $v = r\omega$ gives $\omega = \frac{v}{r} = \frac{5.00\ \text{m/s}}{0.370\ \text{m}} = 13.51\ \text{rad/s}$.

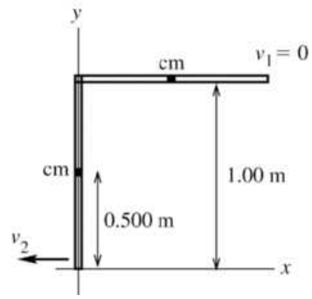
$K = \frac{1}{2}I\omega^2$; therefore $I = \frac{2K}{\omega^2} = \frac{2(56.8\ \text{J})}{(13.51\ \text{rad/s})^2} = 0.622\ \text{kg}\cdot\text{m}^2$.

(b) The wheel's mass is $(280\ \text{N})/(9.8\ \text{m/s}^2) = 28.6\ \text{kg}$. The wheel with the largest possible moment of inertia would have all this mass concentrated in its rim. Its moment of inertia would be $I = MR^2 = (28.6\ \text{kg})(0.370\ \text{m})^2 = 3.92\ \text{kg}\cdot\text{m}^2$. The boss's wheel is physically impossible.

EVALUATE: If the mass falls from rest in free fall its speed after it has descended 2.00 m is $v = \sqrt{2g(2.00\ \text{m})} = 6.26\ \text{m/s}$. Its actual speed is less because some of the energy of the system is in the form of rotational kinetic energy of the wheel.

9.81. **IDENTIFY:** Use conservation of energy. The stick rotates about a fixed axis so $K = \frac{1}{2}I\omega^2$. Once we have ω use $v = r\omega$ to calculate v for the end of the stick.

SET UP: The object is sketched in Figure 9.81.



Take the origin of coordinates at the lowest point reached by the stick and take the positive *y*-direction to be upward.

Figure 9.81

EXECUTE: **(a)** Use Eq.(9.18): $U = Mgy_{cm}$. $\Delta U = U_2 - U_1 = Mg(y_{cm2} - y_{cm1})$. The center of mass of the meter stick is at its geometrical center, so $y_{cm1} = 1.00$ m and $y_{cm2} = 0.50$ m. Then

$\Delta U = (0.180\text{ kg})(9.80\text{ m/s}^2)(0.50\text{ m} - 1.00\text{ m}) = -0.882$ J.

(b) Use conservation of energy: $K_1 + U_1 + W_{other} = K_2 + U_2$. Gravity is the only force that does work on the meter stick, so $W_{other} = 0$. $K_1 = 0$. Thus $K_2 = U_1 - U_2 = -\Delta U$, where ΔU was calculated in part (a). $K_2 = \frac{1}{2}I\omega_2^2$ so $\frac{1}{2}I\omega_2^2 = -\Delta U$ and $\omega_2 = \sqrt{2(-\Delta U)/I}$. For stick pivoted about one end, $I = \frac{1}{3}ML^2$ where

$L = 1.00$ m, so $\omega_2 = \sqrt{\dfrac{6(-\Delta U)}{ML^2}} = \sqrt{\dfrac{6(0.882\text{ J})}{(0.180\text{ kg})(1.00\text{ m})^2}} = 5.42$ rad/s.

(c) $v = r\omega = (1.00\text{ m})(5.42\text{ rad/s}) = 5.42$ m/s.

(d) For a particle in free fall, with $+y$ upward, $v_{0y} = 0$; $y - y_0 = -1.00$ m; $a_y = -9.80\text{ m/s}^2$; and $v_y = ?$ Solving the equation $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ for v_y gives

$v_y = -\sqrt{2a_y(y - y_0)} = -\sqrt{2(-9.80\text{ m/s}^2)(-1.00\text{ m})} = -4.43$ m/s.

EVALUATE: The magnitude of the answer in part (c) is larger. $U_{1,grav}$ is the same for the stick as for a particle falling from a height of 1.00 m. For the stick $K = \frac{1}{2}I\omega_2^2 = \frac{1}{2}(\frac{1}{3}ML^2)(v/L)^2 = \frac{1}{6}Mv^2$. For the stick and for the particle, K_2 is the same but the same *K* gives a larger *v* for the end of the stick than for the particle. The reason is that all the other points along the stick are moving slower than the end opposite the axis.

9.82. **IDENTIFY:** Apply conservation of energy to the system of cylinder and rope.

SET UP: Taking the zero of gravitational potential energy to be at the axle, the initial potential energy is zero (the rope is wrapped in a circle with center on the axle).When the rope has unwound, its center of mass is a distance πR below the axle, since the length of the rope is $2\pi R$ and half this distance is the position of the center of the mass. Initially, every part of the rope is moving with speed $\omega_0 R$, and when the rope has unwound, and the cylinder has angular speed ω, the speed of the rope is ωR (the upper end of the rope has the same tangential speed at the edge of the cylinder). $I = (1/2)MR^2$ for a uniform cylinder.

EXECUTE: $K_1 = K_2 + U_2$. $\left(\dfrac{M}{4} + \dfrac{m}{2}\right)R^2\omega_0^2 = \left(\dfrac{M}{4} + \dfrac{m}{2}\right)R^2\omega^2 - mg\pi R$. Solving for ω gives

$\omega = \sqrt{\omega_0^2 + \dfrac{(4\pi mg/R)}{(M + 2m)}}$, and the speed of any part of the rope is $v = \omega R$.

EVALUATE: When $m \to 0$, $\omega \to \omega_0$, When $m >> M$, $\omega = \sqrt{\omega_0^2 + \dfrac{2\pi g}{R}}$ and $v = \sqrt{v_0^2 + 2\pi gR}$. This is the final speed when an object with initial speed v_0 descends a distance πR.

9.83. **IDENTIFY:** Apply conservation of energy to the system consisting of blocks *A* and *B* and the pulley.
SET UP: The system at points 1 and 2 of its motion is sketched in Figure 9.83.

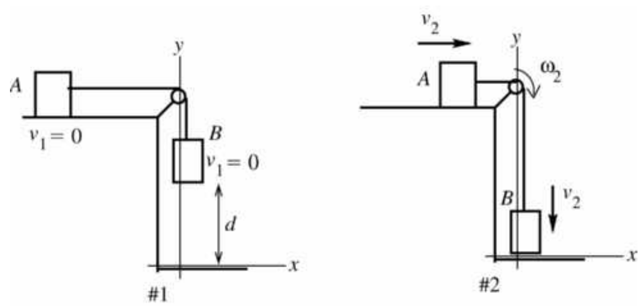


Figure 9.83

Use the work-energy relation $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Use coordinates where $+y$ is upward and where the origin is at the position of block *B* after it has descended. The tension in the rope does positive work on block *A* and negative work of the same magnitude on block *B*, so the net work done by the tension in the rope is zero. Both blocks have the same speed.
EXECUTE: Gravity does work on block *B* and kinetic friction does work on block *A*. Therefore $W_{\text{other}} = W_f = -\mu_k m_A g d$.

$K_1 = 0$ (system is released from rest)

$U_1 = m_B g y_{B1} = m_B g d$; $U_2 = m_B g y_{B2} = 0$

$K_2 = \frac{1}{2} m_A v_2^2 + \frac{1}{2} m_B v_2^2 + \frac{1}{2} I \omega_2^2$.

But $v(\text{blocks}) = R\omega(\text{pulley})$, so $\omega_2 = v_2/R$ and

$K_2 = \frac{1}{2}(m_A + m_B)v_2^2 + \frac{1}{2} I (v_2/R)^2 = \frac{1}{2}(m_A + m_B + I/R^2)v_2^2$

Putting all this into the work-energy relation gives

$m_B g d - \mu_k m_A g d = \frac{1}{2}(m_A + m_B + I/R^2)v_2^2$

$(m_A + m_B + I/R^2)v_2^2 = 2gd(m_B - \mu_k m_A)$

$$v_2 = \sqrt{\frac{2gd(m_B - \mu_k m_A)}{m_A + m_B + I/R^2}}$$

EVALUATE: If $m_B >> m_A$ and I/R^2, then $v_2 = \sqrt{2gd}$; block *B* falls freely. If *I* is very large, v_2 is very small. Must have $m_B > \mu_k m_A$ for motion, so the weight of *B* will be larger than the friction force on *A*. I/R^2 has units of mass and is in a sense the "effective mass" of the pulley.

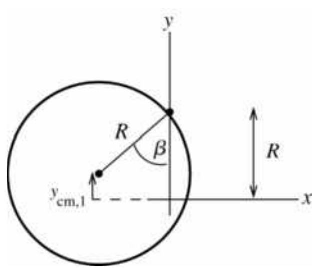
9.84. **IDENTIFY:** Apply conservation of energy to the system of two blocks and the pulley.
SET UP: Let the potential energy of each block be zero at its initial position. The kinetic energy of the system is the sum of the kinetic energies of each object. $v = R\omega$, where *v* is the common speed of the blocks and ω is the angular velocity of the pulley.
EXECUTE: The amount of gravitational potential energy which has become kinetic energy is $K = (4.00\text{ kg} - 2.00\text{ kg})(9.80\text{ m/s}^2)(5.00\text{ m}) = 98.0\text{ J}$. In terms of the common speed *v* of the blocks, the kinetic energy of the system is $K = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2} I\left(\frac{v}{R}\right)^2$.

$K = v^2 \frac{1}{2}\left(4.00\text{ kg} + 2.00\text{ kg} + \frac{(0.560\text{ kg}\cdot\text{m}^2)}{(0.160\text{ m})^2}\right) = v^2(13.94\text{ kg})$. Solving for v gives

$v = \sqrt{\frac{98.0\text{ J}}{13.94\text{ kg}}} = 2.65\text{ m/s}.$

EVALUATE: If the pulley is massless, $98.0\text{ J} = \frac{1}{2}(4.00\text{ kg} + 2.00\text{ kg})v^2$ and $v = 5.72\text{ m/s}$. The moment of inertia of the pulley reduces the final speed of the blocks.

9.85. **IDENTIFY** and **SET UP:** Apply conservation of energy to the motion of the hoop. Use Eq. (9.18) to calculate U_{grav}. Use $K = \frac{1}{2}I\omega^2$ for the kinetic energy of the hoop. Solve for ω. The center of mass of the hoop is at its geometrical center.



Take the origin to be at the original location of the center of the hoop, before it is rotated to one side, as shown in Figure 9.85.

Figure 9.85

$y_{\text{cm1}} = R - R\cos\beta = R(1-\cos\beta)$

$y_{\text{cm2}} = 0$ (at equilibrium position hoop is at original position)

EXECUTE: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$W_{\text{other}} = 0$ (only gravity does work)

$K_1 = 0$ (released from rest), $K_2 = \frac{1}{2}I\omega_2^2$

For a hoop, $I_{\text{cm}} = MR^2$, so $I = Md^2 + MR^2$ with $d = R$ and $I = 2MR^2$, for an axis at the edge. Thus

$K_2 = \frac{1}{2}(2MR^2)\omega_2^2 = MR^2\omega_2^2.$

$U_1 = Mgy_{\text{cm1}} = MgR(1-\cos\beta)$, $U_2 = mgy_{\text{cm2}} = 0$

Thus $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives

$MgR(1-\cos\beta) = MR^2\omega_2^2$ and $\omega_2 = \sqrt{g(1-\cos\beta)/R}$

EVALUATE: If $\beta = 0$, then $\omega_2 = 0$. As β increases, ω_2 increases.

9.86. **IDENTIFY:** $K = \frac{1}{2}I\omega^2$, with ω in rad/s. $P = \frac{\text{energy}}{t}$

SET UP: For a solid cylinder, $I = \frac{1}{2}MR^2$. $1\text{ rev/min} = (2\pi/60)\text{ rad/s}$

EXECUTE: **(a)** $\omega = 3000\text{ rev/min} = 314\text{ rad/s}$. $I = \frac{1}{2}(1000\text{ kg})(0.900\text{ m})^2 = 405\text{ kg}\cdot\text{m}^2$

$K = \frac{1}{2}(405\text{ kg}\cdot\text{m}^2)(314\text{ rad/s})^2 = 2.00\times10^7\text{ J}.$

(b) $t = \frac{K}{P} = \frac{2.00\times10^7\text{ J}}{1.86\times10^4\text{ W}} = 1.08\times10^3\text{ s} = 17.9\text{ min}.$

EVALUATE: In $K = \frac{1}{2}I\omega^2$, we must use ω in rad/s.

9.87. **IDENTIFY:** $I = I_1 + I_2$. Apply conservation of energy to the system. The calculation is similar to Example 9.8.

SET UP: $\omega = \frac{v}{R_1}$ for part (b) and $\omega = \frac{v}{R_2}$ for part (c).

EXECUTE: **(a)** $I=\frac{1}{2}M_1R_1^2+\frac{1}{2}M_2R_2^2=\frac{1}{2}((0.80\text{ kg})(2.50\times10^{-2}\text{ m})^2+(1.60\text{ kg})(5.00\times10^{-2}\text{ m})^2)$

$I=2.25\times10^{-3}\text{ kg}\cdot\text{m}^2$.

(b) The method of Example 9.8 yields $v=\sqrt{\dfrac{2gh}{1+(I/mR_1^2)}}$.

$$v=\sqrt{\frac{2(9.80\text{ m/s}^2)(2.00\text{ m})}{(1+((2.25\times10^{-3}\text{ kg}\cdot\text{m}^2)/(1.50\text{ kg})(0.025\text{ m})^2))}}=3.40\text{ m/s}.$$

(c) The same calculation, with R_2 instead of R_1 gives $v=4.95\text{ m/s}$.

EVALUATE: The final speed of the block is greater when the string is wrapped around the larger disk. $v=R\omega$, so when $R=R_2$ the factor that relates v to ω is larger. For $R=R_2$ a larger fraction of the total kinetic energy resides with the block. The total kinetic energy is the same in both cases (equal to mgh), so when $R=R_2$ the kinetic energy and speed of the block are greater.

9.88. **IDENTIFY:** The potential energy of the falling block is transformed into kinetic energy of the block and kinetic energy of the turning wheel, but some of it is lost to the work by friction. Energy conservation applies, with the target variable being the angular velocity of the wheel when the block has fallen a given distance.

SET UP: $K_1+U_1+W_{\text{other}}=K_2+U_2$, where $K=\frac{1}{2}mv^2$, $U=mgh$, and W_{other} is the work done by friction.

EXECUTE: Energy conservation gives $mgh+(-6.00\text{ J})=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2$. $v=R\omega$, so $\frac{1}{2}mv^2=\frac{1}{2}mR^2\omega^2$ and $mgh+(-6.00\text{ J})=\frac{1}{2}(mR^2+I)\omega^2$. Solving for ω gives

$$\omega=\sqrt{\frac{2[mgh+(-6.00\text{ J})]}{mR^2+I}}=\sqrt{\frac{2[(0.340\text{ kg})(9.8\text{ m/s}^2)(3.00\text{ m})-6.00\text{ J}]}{(0.340\text{ kg})(0.180\text{ m})^2+0.480\text{ kg}\cdot\text{m}^2}}=4.03\text{ rad/s}.$$

EVALUATE: Friction does negative work because it opposes the turning of the wheel.

9.89. **IDENTIFY:** Apply conservation of energy to relate the height of the mass to the kinetic energy of the cylinder.

SET UP: First use $K(\text{cylinder})=480\text{ J}$ to find ω for the cylinder and v for the mass.

EXECUTE: $I=\frac{1}{2}MR^2=\frac{1}{2}(10.0\text{ kg})(0.150\text{ m})^2=0.1125\text{ kg}\cdot\text{m}^2$. $K=\frac{1}{2}I\omega^2$ so $\omega=\sqrt{2K/I}=92.38\text{ rad/s}$. $v=R\omega=13.86\text{ m/s}$.

SET UP: Use conservation of energy $K_1+U_1=K_2+U_2$ to solve for the distance the mass descends. Take $y=0$ at lowest point of the mass, so $y_2=0$ and $y_1=h$, the distance the mass descends.

EXECUTE: $K_1=U_2=0$ so $U_1=K_2$. $mgh=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2$, where $m=12.0\text{ kg}$. For the cylinder, $I=\frac{1}{2}MR^2$ and $\omega=v/R$, so $\frac{1}{2}I\omega^2=\frac{1}{4}Mv^2$. Solving $mgh=\frac{1}{2}mv^2+\frac{1}{4}Mv^2$ for h gives

$$h=\frac{v^2}{2g}\left(1+\frac{M}{2m}\right)=13.9\text{ m}.$$

EVALUATE: For the cylinder $K_{\text{cyl}}=\frac{1}{2}I\omega^2=\frac{1}{2}(\frac{1}{2}MR^2)(v/R)^2=\frac{1}{4}Mv^2$. $K_{\text{mass}}=\frac{1}{2}mv^2$, so $K_{\text{mass}}=(2m/M)K_{\text{cyl}}=[2(12.0\text{ kg})/10.0\text{ kg}](480\text{ J})=1150\text{ J}$. The mass has 1150 J of kinetic energy when the cylinder has 480 J of kinetic energy and at this point the system has total energy 1630 J since $U_2=0$. Initially the total energy of the system is $U_1=mgy_1=mgh=1630\text{ J}$, so the total energy is shown to be conserved.

9.90. IDENTIFY: Energy conservation: Loss of U of box equals gain in K of system. Both the cylinder and pulley have kinetic energy of the form $K = \frac{1}{2}I\omega^2$.

$$m_{\text{box}}gh = \frac{1}{2}m_{\text{box}}v_{\text{box}}^2 + \frac{1}{2}I_{\text{pulley}}\omega_{\text{pulley}}^2 + \frac{1}{2}I_{\text{cylinder}}\omega_{\text{cylinder}}^2.$$

SET UP: $\omega_{\text{pulley}} = \dfrac{v_{\text{box}}}{r_{\text{p}}}$ and $\omega_{\text{cylinder}} = \dfrac{v_{\text{box}}}{r_{\text{cylinder}}}$.

Let B = box, P = pulley and C = cylinder.

EXECUTE: $m_{\text{B}}gh = \frac{1}{2}m_{\text{B}}v_{\text{B}}^2 + \frac{1}{2}\left(\frac{1}{2}m_{\text{P}}r_{\text{P}}^2\right)\left(\frac{v_{\text{B}}}{r_{\text{P}}}\right)^2 + \frac{1}{2}\left(\frac{1}{2}m_{\text{C}}r_{\text{C}}^2\right)\left(\frac{v_{\text{B}}}{r_{\text{C}}}\right)^2$.

$m_{\text{B}}gh = \frac{1}{2}m_{\text{B}}v_{\text{B}}^2 + \frac{1}{4}m_{\text{P}}v_{\text{B}}^2 + \frac{1}{4}m_{\text{C}}v_{\text{B}}^2$ and

$$v_{\text{B}} = \sqrt{\frac{m_{\text{B}}gh}{\frac{1}{2}m_{\text{B}} + \frac{1}{4}m_{\text{P}} + \frac{1}{4}m_{\text{C}}}} = \sqrt{\frac{(3.00\text{ kg})(9.80\text{ m/s}^2)(2.50\text{ m})}{1.50\text{ kg} + \frac{1}{4}(7.00\text{ kg})}} = 4.76\text{ m/s}.$$

EVALUATE: If the box was disconnected from the rope and dropped from rest, after falling 2.50 m its speed would be $v = \sqrt{2g(2.50\text{ m})} = 7.00\text{ m/s}$. Since in the problem some of the energy of the system goes into kinetic energy of the cylinder and of the pulley, the final speed of the box is less than this.

9.91. IDENTIFY: $I = I_{\text{disk}} - I_{\text{hole}}$, where I_{hole} is I for the piece punched from the disk. Apply the parallel-axis theorem to calculate the required moments of inertia.

SET UP: For a uniform disk, $I = \frac{1}{2}MR^2$.

EXECUTE: **(a)** The initial moment of inertia is $I_0 = \frac{1}{2}MR^2$. The piece punched has a mass of $\dfrac{M}{16}$ and a moment of inertia with respect to the axis of the original disk of

$$\frac{M}{16}\left[\frac{1}{2}\left(\frac{R}{4}\right)^2 + \left(\frac{R}{2}\right)^2\right] = \frac{9}{512}MR^2.$$

The moment of inertia of the remaining piece is then $I = \dfrac{1}{2}MR^2 - \dfrac{9}{512}MR^2 = \dfrac{247}{512}MR^2$.

(b) $I = \frac{1}{2}MR^2 + M(R/2)^2 - \frac{1}{2}(M/16)(R/4)^2 = \frac{383}{512}MR^2$.

EVALUATE: For a solid disk and an axis at a distance $R/2$ from the disk's center, the parallel-axis theorem gives $I = \frac{1}{2}MR^2 = \frac{3}{4}MR^2 = \frac{384}{512}MR^2$. For both choices of axes the presence of the hole reduces I, but the effect of the hole is greater in part (a), when it is farther from the axis.

9.92. IDENTIFY: We know (or can calculate) the masses and geometric measurements of the various parts of the body. We can model them as familiar objects, such as uniform spheres, rods, and cylinders, and calculate their moments of inertia and kinetic energies.

SET UP: My total mass is $m = 90$ kg. I model my head as a uniform sphere of radius 8 cm. I model my trunk and legs as a uniform solid cylinder of radius 12 cm. I model my arms as slender rods of length 60 cm. $\omega = 72$ rev/min = 7.5 rad/s. For a solid uniform sphere, $I = 2/5\ MR^2$, for a solid cylinder, $I = \frac{1}{2}MR^2$, and for a rod rotated about one end $I = 1/3\ ML^2$.

EXECUTE: **(a)** Using the formulas indicated above, we have $I_{\text{tot}} = I_{\text{head}} + I_{\text{trunk+legs}} + I_{\text{arms}}$, which gives $I_{\text{tot}} = \frac{2}{5}(0.070m)(0.080\text{ m})^2 + \frac{1}{2}(0.80m)(0.12\text{ m})^2 + 2\left(\frac{1}{3}\right)(0.13m)(0.60\text{ m})^2 = 3.3\text{ kg}\cdot\text{m}^2$ where we have used $m = 90$ kg.

(b) $K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(3.3\text{ kg}\cdot\text{m}^2)(7.5\text{ rad/s})^2 = 93\text{ J}$.

EVALUATE: According to these estimates about 85% of the total I is due to the outstretched arms. If the initial translational kinetic energy $\frac{1}{2}mv^2$ of the skater is converted to this rotational kinetic energy as he goes into a spin, his initial speed must be 1.4 m/s.

9.93. **IDENTIFY:** The total kinetic energy of a walker is the sum of his translational kinetic energy plus the rotational kinetic of his arms and legs. We can model these parts of the body as uniform bars.

SET UP: For a uniform bar pivoted about one end, $I = \frac{1}{3}mL^2$. $v = 5.0\text{ km/h} = 1.4\text{ m/s}$.

$K_{\text{tran}} = \frac{1}{2}mv^2$ and $K_{\text{rot}} = \frac{1}{2}I\omega^2$.

EXECUTE: **(a)** $60° = \left(\frac{1}{3}\right)$ rad. The average angular speed of each arm and leg is $\dfrac{\frac{1}{3}\text{ rad}}{1\text{ s}} = 1.05\text{ rad/s}$.

(b) Adding the moments of inertia gives

$I = \frac{1}{3}m_{\text{arm}}L_{\text{arm}}{}^2 + \frac{1}{3}m_{\text{leg}}L_{\text{leg}}{}^2 = \frac{1}{3}[(0.13)(75\text{ kg})(0.70\text{ m})^2 + (0.37)(75\text{ kg})(0.90\text{ m})^2]$. $I = 9.08\text{ kg}\cdot\text{m}^2$.

$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(9.08\text{ kg}\cdot\text{m}^2)(1.05\text{ rad/s})^2 = 5.0\text{ J}$.

(c) $K_{\text{tran}} = \frac{1}{2}mv^2 = \frac{1}{2}(75\text{ kg})(1.4\text{ m/s})^2 = 73.5\text{ J}$ and $K_{\text{tot}} = K_{\text{tran}} + K_{\text{rot}} = 78.5\text{ J}$.

(d) $\dfrac{K_{\text{rot}}}{K_{\text{tran}}} = \dfrac{5.0\text{ J}}{78.5\text{ J}} = 6.4\%$.

EVALUATE: If you swing your arms more vigorously more of your energy input goes into the kinetic energy of walking and it is more effective exercise. Carrying weights in our hands would also be effective.

9.94. **IDENTIFY:** The total kinetic energy of a runner is the sum of his translational kinetic energy plus the rotational kinetic of his arms and legs. We can model these parts of the body as uniform bars.

SET UP: Now $v = 12\text{ km/h} = 3.33\text{ m/s}$. $I_{\text{tot}} = 9.08\text{ kg}\cdot\text{m}^2$ as in Problem 9.93.

EXECUTE: **(a)** $\omega_{\text{av}} = \dfrac{1/3\text{ rad}}{0.5\text{ s}} = 2.1\text{ rad/s}$.

(b) $K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(9.08\text{ kg}\cdot\text{m}^2)(2.1\text{ rad/s})^2 = 20\text{ J}$.

(c) $K_{\text{tran}} = \frac{1}{2}mv^2 = \frac{1}{2}(75\text{ kg})(3.33\text{ m/s})^2 = 416\text{ J}$. Therefore

$K_{\text{tot}} = K_{\text{tran}} + K_{\text{rot}} = 416\text{ J} + 20\text{ J} = 436\text{ J}$.

(d) $\dfrac{K_{\text{rot}}}{K_{\text{tot}}} = \dfrac{20\text{ J}}{436\text{ J}} = 4.6\%$.

9.95. **IDENTIFY:** Follow the instructions in the problem to derive the perpendicular-axis theorem. Then apply that result in part (b).

SET UP: $I = \sum_i m_i r_i^2$. The moment of inertia for the washer and an axis perpendicular to the plane of the washer at its center is $\frac{1}{2}M(R_1^2 + R_2^2)$. In part (b), I for an axis perpendicular to the plane of the square at its center is $\frac{1}{12}M\left(L^2 + L^2\right) = \frac{1}{6}ML^2$.

EXECUTE: **(a)** With respect to O, $r_i^2 = x_i^2 + y_i^2$, and so

$I_O = \sum_i m_i r_i^2 = \sum_i m_i(x_i^2 + y_i^2) = \sum_i m_i x_i^2 + \sum_i m_i y_i^2 = I_x + I_y$.

(b) Two perpendicular axes, both perpendicular to the washer's axis, will have the same moment of inertia about those axes, and the perpendicular-axis theorem predicts that they will sum to the moment of inertia about the washer axis, which is $I = \frac{1}{2}M(R_1^2 + R_2^2)$, and so $I_x = I_y = \frac{1}{4}M(R_1^2 + R_2^2)$.

(c) $I_0 = \frac{1}{6}mL^2$. Since $I_0 = I_x + I_y$, and $I_x = I_y$, both I_x and I_y must be $\frac{1}{12}mL^2$.

EVALUATE: The result in part (c) says that I is the same for an axis that bisects opposite sides of the square as for an axis along the diagonal of the square, even though the distribution of mass relative to the two axes is quite different in these two cases.

9.96. **IDENTIFY:** Apply the parallel-axis theorem to each side of the square.

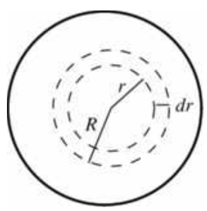
SET UP: Each side has length a and mass $M/4$, and the moment of inertia of each side about an axis perpendicular to the side and through its center is $\frac{1}{12}\left(\frac{1}{4}Ma^2\right) = \frac{1}{48}Ma^2$.

EXECUTE: The moment of inertia of each side about the axis through the center of the square is, from the perpendicular axis theorem, $\frac{Ma^2}{48}+\frac{M}{4}\left(\frac{a}{2}\right)^2=\frac{Ma^2}{12}$. The total moment of inertia is the sum of the contributions from the four sides, or $4\times\frac{Ma^2}{12}=\frac{Ma^2}{3}$.

EVALUATE: If all the mass of a side were at its center, a distance $a/2$ from the axis, we would have $I=4\left(\frac{M}{4}\right)\left(\frac{a}{2}\right)^2=\frac{1}{4}Ma^2$. If all the mass was divided equally among the four corners of the square, a distance $a/\sqrt{2}$ from the axis, we would have $I=4\left(\frac{M}{4}\right)\left(\frac{a}{\sqrt{2}}\right)^2=\frac{1}{2}Ma^2$. The actual I is between these two values.

9.97. **IDENTIFY:** Use Eq. (9.20) to calculate I.

(a) SET UP: Let L be the length of the cylinder. Divide the cylinder into thin cylindrical shells of inner radius r and outer radius $r+dr$. An end view is shown in Figure 9.97



$\rho=\alpha r$

The mass of the thin cylindrical shell is

$dm=\rho\, dV=\rho(2\pi r\, dr)L=2\pi\alpha L r^2\, dr$

Figure 9.97

EXECUTE: $I=\int r^2\, dm=2\pi\alpha L\int_0^R r^4\, dr=2\pi\alpha L\left(\frac{1}{5}R^5\right)=\frac{2}{5}\pi\alpha L R^5$

Relate M to α: $M=\int dm=2\pi\alpha L\int_0^R r^2\, dr=2\alpha\pi L\left(\frac{1}{3}R^3\right)=\frac{2}{3}\pi\alpha L R^3$, so $\pi\alpha L R^3=3M/2$.

Using this in the above result for I gives $I=\frac{2}{5}(3M/2)R^2=\frac{3}{5}MR^2$.

(b) EVALUATE: For a cylinder of uniform density $I=\frac{1}{2}MR^2$. The answer in (a) is larger than this. Since the density increases with distance from the axis the cylinder in (a) has more mass farther from the axis than for a cylinder of uniform density.

9.98. **IDENTIFY:** Write K in terms of the period T and take derivatives of both sides of this equation to relate dK/dt to dT/dt.

SET UP: $\omega=\frac{2\pi}{T}$ and $K=\frac{1}{2}I\omega^2$. The speed of light is $c=3.00\times10^8$ m/s.

EXECUTE: **(a)** $K=\frac{2\pi^2 I}{T^2}$. $\frac{dK}{dt}=-\frac{4\pi^2 I}{T^3}\frac{dT}{dt}$. The rate of energy loss is $\frac{4\pi^2 I}{T^3}\frac{dT}{dt}$. Solving for the moment of inertia I in terms of the power P,

$$I=\frac{PT^3}{4\pi^2}\frac{1}{dT/dt}=\frac{(5\times10^{31}\text{ W})(0.0331\text{ s})^3}{4\pi^2}\frac{1\text{ s}}{4.22\times10^{-13}\text{ s}}=1.09\times10^{38}\text{ kg}\cdot\text{m}^2$$

(b) $R=\sqrt{\frac{5I}{2M}}=\sqrt{\frac{5(1.08\times10^{38}\text{ kg}\cdot\text{m}^2)}{2(1.4)(1.99\times10^{30}\text{ kg})}}=9.9\times10^3$ m, about 10 km.

(c) $v=\frac{2\pi R}{T}=\frac{2\pi(9.9\times10^3\text{m})}{(0.0331\text{ s})}=1.9\times10^6\text{ m/s}=6.3\times10^{-3}c$.

(d) $\rho = \dfrac{M}{V} = \dfrac{M}{(4\pi/3)R^3} = 6.9\times 10^{17}\text{ kg/m}^3$, which is much higher than the density of ordinary rock by 14 orders of magnitude, and is comparable to nuclear mass densities.

EVALUATE: I is huge because M is huge. A small rate of change in the period corresponds to a large release of energy.

9.99. **IDENTIFY:** The density depends on the distance from the center of the sphere, so it is a function of r. We need to integrate to find the mass and the moment of inertia.

SET UP: $M = \int dm = \int \rho dV$ and $I = \int dI$.

EXECUTE: **(a)** Divide the sphere into thin spherical shells of radius r and thickness dr. The volume of each shell is $dV = 4\pi r^2 dr$. $\rho(r) = a - br$, with $a = 3.00\times 10^3\text{ kg/m}^3$ and $b = 9.00\times 10^3\text{ kg/m}^4$. Integrating gives $M = \int dm = \int \rho dV = \int_0^R (a-br)4\pi r^2 dr = \frac{4}{3}\pi R^3\left(a - \frac{3}{4}bR\right)$.

$$M = \frac{4}{3}\pi(0.200)^3\left(3.00\times 10^3\text{ kg/m}^3 - \frac{3}{4}(9.00\times 10^3\text{ kg/m}^4)(0.200\text{ m})\right) = 55.3\text{ kg}.$$

(b) The moment of inertia of each thin spherical shell is

$$dI = \frac{2}{3}r^2 dm = \frac{2}{3}r^2\rho dV = \frac{2}{3}r^2(a-br)4\pi r^2 dr = \frac{8\pi}{3}r^4(a-br)dr.$$

$$I = \int_0^R dI = \frac{8\pi}{3}\int_0^R r^4(a-br)dr = \frac{8\pi}{15}R^5\left(a - \frac{5b}{6}R\right).$$

$$I = \frac{8\pi}{15}(0.200\text{ m})^5\left(3.00\times 10^3\text{ kg/m}^3 - \frac{5}{6}(9.00\times 10^3\text{ kg/m}^4)(0.200\text{ m})\right) = 0.804\text{ kg}\cdot\text{m}^2.$$

EVALUATE: We cannot use the formulas $M = \rho V$ and $I = \frac{1}{2}MR^2$ because this sphere is not uniform throughout. Its density increases toward the surface. For a uniform sphere with density $3.00\times 10^3\text{ kg/m}^3$, the mass is $\frac{4}{3}\pi R^3\rho = 100.5\text{ kg}$. The mass of the sphere in this problem is less than this. For a uniform sphere with mass 55.3 kg and $R = 0.200$ m, $I = \frac{2}{5}MR^2 = 0.885\text{ kg}\cdot\text{m}^2$. The moment of inertia for the sphere in this problem is less than this, since the density decreases with distance from the center of the sphere.

9.100. **IDENTIFY:** Apply Eq. (9.20).

SET UP: Let z be the coordinate along the vertical axis. $r(z) = \dfrac{zR}{h}$. $dm = \pi\rho\dfrac{R^2z^2}{h^2}$ and $dI = \dfrac{\pi\rho}{2}\dfrac{R^4}{h^4}z^4\,dz$.

EXECUTE: $I = \int dI = \dfrac{\pi\rho}{2}\dfrac{R^4}{h^4}\int_0^h z^4\,dz = \dfrac{\pi\rho}{10}\dfrac{R^4}{h^4}\left[z^5\right]_0^h = \dfrac{1}{10}\pi\rho R^4 h$. The volume of a right circular cone is $V = \frac{1}{3}\pi R^2 h$, the mass is $\frac{1}{3}\pi\rho R^2 h$ and so $I = \dfrac{3}{10}\left(\dfrac{\pi\rho R^2 h}{3}\right)R^2 = \dfrac{3}{10}MR^2$.

EVALUATE: For a uniform cylinder of radius R and for an axis through its center, $I = \frac{1}{2}MR^2$. I for the cone is less, as expected, since the cone is constructed from a series of parallel discs whose radii decrease from R to zero along the vertical axis of the cone.

9.101. **IDENTIFY:** Follow the steps outlined in the problem.

SET UP: $\omega_z = d\theta/dt$. $\alpha_z = d^2\omega_z/dt^2$.

EXECUTE: **(a)** $ds = r\,d\theta = r_0 d\theta + \beta\theta\,d\theta$ so $s(\theta) = r_0\theta + \frac{\beta}{2}\theta^2$. θ must be in radians.

(b) Setting $s = vt = r_0\theta + \frac{\beta}{2}\theta^2$ gives a quadratic in θ. The positive solution is

$\theta(t) = \frac{1}{\beta}\left[\sqrt{r_0{}^2 + 2\beta vt} - r_0\right].$

(The negative solution would be going backwards, to values of r smaller than r_0.)

(c) Differentiating, $\omega_z(t) = \frac{d\theta}{dt} = \frac{v}{\sqrt{r_0^2 + 2\beta vt}}$, $\alpha_z = \frac{d\omega_z}{dt} = -\frac{\beta v^2}{(r_0^2 + 2\beta vt)^{3/2}}$. The angular acceleration α_z is not constant.

(d) $r_0 = 25.0$ mm. θ must be measured in radians, so $\beta = (1.55\,\mu\text{m/rev})(1\text{ rev}/2\pi\text{ rad}) = 0.247\,\mu\text{m/rad}$. Using $\theta(t)$ from part (b), the total angle turned in $74.0\text{ min} = 4440\text{ s}$ is

$$\theta = \frac{1}{2.47\times10^{-7}\text{ m/rad}}\left(\sqrt{2(2.47\times10^{-7}\text{ m/rad})(1.25\text{ m/s})(4440\text{ s}) + (25.0\times10^{-3}\text{m})^2} - 25.0\times10^{-3}\text{ m}\right)$$

$\theta = 1.337\times10^5$ rad, which is 2.13×10^4 rev.

(e) The graphs are sketched in Figure 9.101.

EVALUATE: ω_z must decrease as r increases, to keep $v = r\omega$ constant. For ω_z to decrease in time, α_z must be negative.

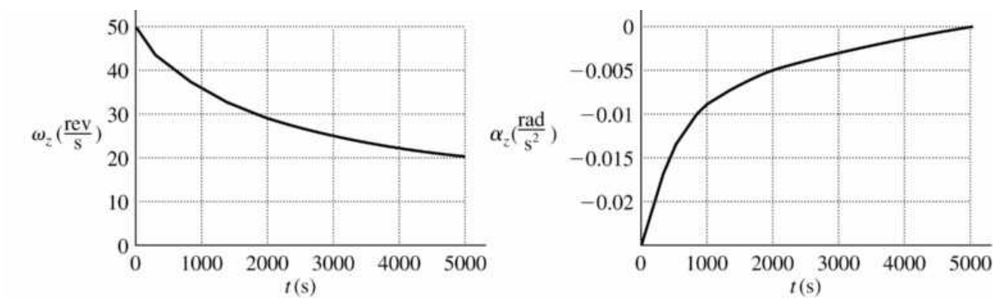


Figure 9.101