13

GRAVITATION

13.1. **IDENTIFY and SET UP:** Use the law of gravitation, Eq. (13.1), to determine F_g.

EXECUTE: $F_{\text{S on M}} = G\dfrac{m_S m_M}{r_{SM}^2}$ (S = sun, M = moon); $F_{\text{E on M}} = G\dfrac{m_E m_M}{r_{EM}^2}$ (E = earth)

$$\frac{F_{\text{S on M}}}{F_{\text{E on M}}} = \left(G\frac{m_S m_M}{r_{SM}^2}\right)\left(\frac{r_{EM}^2}{G m_E m_M}\right) = \frac{m_S}{m_E}\left(\frac{r_{EM}}{r_{SM}}\right)^2$$

r_{EM}, the radius of the moon's orbit around the earth is given in Appendix F as 3.84×10^8 m. The moon is much closer to the earth than it is to the sun, so take the distance r_{SM} of the moon from the sun to be r_{SE}, the radius of the earth's orbit around the sun.

$$\frac{F_{\text{S on M}}}{F_{\text{E on M}}} = \left(\frac{1.99\times10^{30}\ \text{kg}}{5.98\times10^{24}\ \text{kg}}\right)\left(\frac{3.84\times10^8\ \text{m}}{1.50\times10^{11}\ \text{m}}\right)^2 = 2.18.$$

EVALUATE: The force exerted by the sun is larger than the force exerted by the earth. The moon's motion is a combination of orbiting the sun and orbiting the earth.

13.2. **IDENTIFY:** The gravity force between spherically symmetric spheres is $F_g = \dfrac{Gm_1m_2}{r^2}$, where r is the separation between their centers.

SET UP: $G = 6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$. The moment arm for the torque due to each force is 0.150 m.

EXECUTE: **(a)** For each pair of spheres, $F_g = \dfrac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.10\ \text{kg})(25.0\ \text{kg})}{(0.120\ \text{m})^2} = 1.27\times10^{-7}\ \text{N}.$

From Figure 13.4 in the textbook we see that the forces for each pair are in opposite directions, so $F_{net} = 0$.

(b) The net torque is $\tau_{net} = 2F_g l = 2(1.27\times10^{-7}\ \text{N})(0.150\ \text{m}) = 3.81\times10^{-8}\ \text{N}\cdot\text{m}.$

(c) The torque is very small and the apparatus must be very sensitive. The torque could be increased by increasing the mass of the spheres or by decreasing their separation.

EVALUATE: The quartz fiber must twist through a measurable angle when a small torque is applied to it.

13.3. **IDENTIFY:** The gravitational attraction of the astronauts on each other causes them to accelerate toward each other, so Newton's second law of motion applies to their motion.

SET UP: The net force on each astronaut is the gravity force exerted by the other astronaut. Call the astronauts A and B, where $m_A = 65$ kg and $m_B = 72$ kg. $F_{grav} = Gm_1m_2/r^2$ and $\Sigma F = ma$.

EXECUTE: **(a)** The free-body diagram for astronaut A is given in Figure 13.3a and for astronaut B in Figure 13.3b.

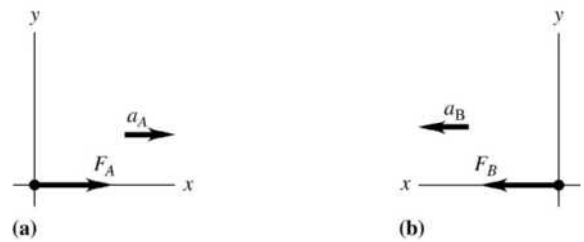


Figure 13.3

$\Sigma F_x = ma_x$ for A gives $F_A = m_A a_A$ and $a_A = \dfrac{F_A}{m_A}$. And for B, $a_B = \dfrac{F_B}{m_B}$.

$$F_A = F_B = G\frac{m_A m_B}{r^2} = (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(65\ \text{kg})(72\ \text{kg})}{(20.0\ \text{m})^2} = 7.807\times10^{-10}\ \text{N} \text{ so}$$

$$a_A = \frac{7.807\times10^{-10}\ \text{N}}{65\ \text{kg}} = 1.2\times10^{-11}\ \text{m/s}^2 \text{ and } a_B = \frac{7.807\times10^{-10}\ \text{N}}{72\ \text{kg}} = 1.1\times10^{-11}\ \text{m/s}^2.$$

(b) Using constant-acceleration kinematics, we have $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$, which gives $x_A = \frac{1}{2}a_A t^2$ and $x_B = \frac{1}{2}a_B t^2$. $x_A + x_B = 20.0$ m, so $20.0\ \text{m} = \frac{1}{2}(a_A + a_B)t^2$ and

$$t = \sqrt{\frac{2(20.0\ \text{m})}{1.2\times10^{-11}\ \text{m/s}^2 + 1.1\times10^{-11}\ \text{m/s}^2}} = 1.32\times10^6\ \text{s} = 15\ \text{days}.$$

(c) Their accelerations would increase as they moved closer and the gravitational attraction between them increased.

EVALUATE: Even though the gravitational attraction of the astronauts is much weaker than ordinary forces on earth, if it were the only force acting on the astronauts, it would produce noticeable effects.

13.4. **IDENTIFY:** Apply Eq. (13.2), generalized to any pair of spherically symmetric objects.

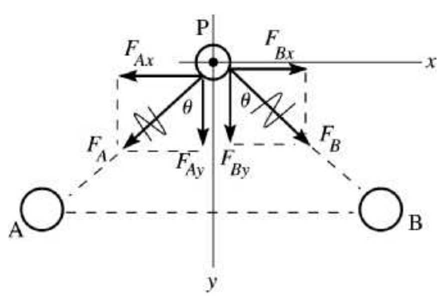
SET UP: The separation of the centers of the spheres is $2R$.

EXECUTE: The magnitude of the gravitational attraction is $GM^2/(2R)^2 = GM^2/4R^2$.

EVALUATE: Eq. (13.2) applies to any pair of spherically symmetric objects; one of the objects doesn't have to be the earth.

13.5. **IDENTIFY:** Use Eq. (13.1) to find the force exerted by each large sphere. Add these forces as vectors to get the net force and then use Newton's 2nd law to calculate the acceleration.

SET UP: The forces are shown in Figure 13.5.



$\sin\theta = 0.80$
$\cos\theta = 0.60$
Take the origin of coordinate at point P.

Figure 13.5

EXECUTE: $F_A = G\dfrac{m_A m}{r^2} = G\dfrac{(0.26\ \text{kg})(0.010\ \text{kg})}{(0.100\ \text{m})^2} = 1.735\times10^{-11}\ \text{N}$

$$F_B = G\frac{m_B m}{r^2} = 1.735\times10^{-11}\ \text{N}$$

$$F_{Ax} = -F_A\sin\theta = -(1.735\times10^{-11}\ \text{N})(0.80) = -1.39\times10^{-11}\ \text{N}$$

$F_{Ay} = +F_A \cos\theta = +(1.735\times10^{-11}\text{ N})(0.60) = +1.04\times10^{-11}\text{ N}$

$F_{Bx} = +F_B \sin\theta = +1.39\times10^{-11}\text{ N}$

$F_{By} = +F_B \cos\theta = +1.04\times10^{-11}\text{ N}$

$\Sigma F_x = ma_x$ gives $F_{Ax} + F_{Bx} = ma_x$

$0 = ma_x$ so $a_x = 0$

$\Sigma F_y = ma_y$ gives $F_{Ay} + F_{By} = ma_y$

$2(1.04\times10^{-11}\text{ N}) = (0.010\text{ kg})a_y$

$a_y = 2.1\times10^{-9}\text{ m/s}^2$, directed downward midway between *A* and *B*

EVALUATE: For ordinary size objects the gravitational force is very small, so the initial acceleration is very small. By symmetry there is no *x*-component of net force and the *y*-component is in the direction of the two large spheres, since they attract the small sphere.

13.6. **IDENTIFY:** The net force on *A* is the vector sum of the force due to *B* and the force due to *C*. In part (a), the two forces are in the same direction, but in (b) they are in opposite directions.

SET UP: Use coordinates where $+x$ is to the right. Each gravitational force is attractive, so is toward the mass exerting it. Treat the masses as uniform spheres, so the gravitational force is the same as for point masses with the same center-to-center distances. The free-body diagrams for (a) and (b) are given in Figures 13.6a and 13.6b. The gravitational force is $F_{\text{grav}} = Gm_1m_2/r^2$.

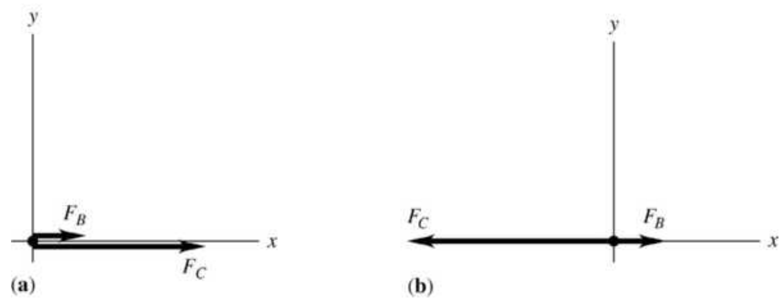


Figure 13.6

EXECUTE: **(a)** Calling F_B the force due to mass *B* and likewise for *C*, we have

$$F_B = G\frac{m_Am_B}{r_{AB}^2} = (6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{(2.00\text{ kg})^2}{(0.50\text{ m})^2} = 1.069\times10^{-9}\text{ N}\ \text{ and}$$

$$F_C = G\frac{m_Am_C}{r_{AC}^2} = (6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{(2.00\text{ kg})^2}{(0.10\text{ m})^2} = 2.669\times10^{-8}\text{ N}.\ \text{ The net force is}$$

$F_{\text{net},\,x} = F_{Bx} + F_{Cx} = 1.069\times10^{-9}\text{ N} + 2.669\times10^{-8}\text{ N} = 2.8\times10^{-8}\text{ N}$ to the right.

(b) Following the same procedure as in (a), we have

$$F_B = G\frac{m_Am_B}{r_{AB}^2} = (6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{(2.00\text{ kg})^2}{(0.40\text{ m})^2} = 1.668\times10^{-9}\text{ N}$$

$$F_C = G\frac{m_Am_C}{r_{AC}^2} = (6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{(2.00\text{ kg})^2}{(0.10\text{ m})^2} = 2.669\times10^{-8}\text{ N}$$

$F_{\text{net},\,x} = F_{Bx} + F_{Cx} = 1.668\times10^{-9}\text{ N} - 2.669\times10^{-8}\text{ N} = -2.5\times10^{-8}\text{ N}$

The net force on *A* is $2.5\times10^{-8}\text{ N}$, to the left.

EVALUATE: As with any force, the gravitational force is a vector and must be treated like all other vectors. The formula $F_{\text{grav}} = Gm_1m_2/r^2$ only gives the magnitude of this force.

13.7. **IDENTIFY:** The force exerted by the moon is the gravitational force, $F_g = \frac{Gm_M m}{r^2}$. The force exerted on the person by the earth is $w = mg$.

SET UP: The mass of the moon is $m_M = 7.35\times10^{22}$ kg. $G = 6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$.

EXECUTE: **(a)** $F_{\text{moon}} = F_g = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(7.35\times10^{22}\ \text{kg})(70\ \text{kg})}{(3.78\times10^8\ \text{m})^2} = 2.4\times10^{-3}$ N.

(b) $F_{\text{earth}} = w = (70\ \text{kg})(9.80\ \text{m/s}^2) = 690$ N. $F_{\text{moon}}/F_{\text{earth}} = 3.5\times10^{-6}$.

EVALUATE: The force exerted by the earth is much greater than the force exerted by the moon. The mass of the moon is less than the mass of the earth and the center of the earth is much closer to the person than is the center of the moon.

13.8. **IDENTIFY:** Use Eq. (13.2) to find the force each point mass exerts on the particle, find the net force, and use Newton's second law to calculate the acceleration.

SET UP: Each force is attractive. The particle (mass m) is a distance $r_1 = 0.200$ m from $m_1 = 8.00$ kg and therefore a distance $r_2 = 0.300$ m from $m_2 = 15.0$ kg. Let $+x$ be toward the 15.0 kg mass.

EXECUTE: $F_1 = \frac{Gm_1 m}{r_1^2} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(8.00\ \text{kg})m}{(0.200\ \text{m})^2} = (1.334\times10^{-8}\ \text{N/kg})m$, in the $-x$-direction. $F_2 = \frac{Gm_2 m}{r_2^2} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(15.0\ \text{kg})m}{(0.300\ \text{m})^2} = (1.112\times10^{-8}\ \text{N/kg})m$, in the $+x$-direction. The net force is

$F_x = F_{1x} + F_{2x} = (-1.334\times10^{-8}\ \text{N/kg} + 1.112\times10^{-8}\ \text{N/kg})m = (-2.2\times10^{-9}\ \text{N/kg})m$.

$a_x = \frac{F_x}{m} = -2.2\times10^{-9}\ \text{m/s}^2$. The acceleration is $2.2\times10^{-9}\ \text{m/s}^2$, toward the 8.00 kg mass.

EVALUATE: The smaller mass exerts the greater force, because the particle is closer to the smaller mass.

13.9. **IDENTIFY:** Use Eq. (13.2) to calculate the gravitational force each particle exerts on the third mass. The equilibrium is stable when for a displacement from equilibrium the net force is directed toward the equilibrium position and it is unstable when the net force is directed away from the equilibrium position.

SET UP: For the net force to be zero, the two forces on *M* must be in opposite directions. This is the case only when *M* is on the line connecting the two particles and between them. The free-body diagram for *M* is given in Figure 13.9. $m_1 = 3m$ and $m_2 = m$. If *M* is a distance *x* from m_1, it is a distance $1.00\ \text{m} - x$ from m_2.

EXECUTE: **(a)** $F_x = F_{1x} + F_{2x} = -G\frac{3mM}{x^2} + G\frac{mM}{(1.00\ \text{m} - x)^2} = 0.\ 3\ (1.00\ \text{m} - x)^2 = x^2$.

$1.00\ \text{m} - x = \pm x/\sqrt{3}$. Since *M* is between the two particles, *x* must be less than 1.00 m and $x = \frac{1.00\ \text{m}}{1 + 1/\sqrt{3}} = 0.634$ m. *M* must be placed at a point that is 0.634 m from the particle of mass $3m$ and 0.366 m from the particle of mass *m*.

(b) (i) If *M* is displaced slightly to the right in Figure 13.9, the attractive force from *m* is larger than the force from $3m$ and the net force is to the right. If *M* is displaced slightly to the left in Figure 13.9, the attractive force from $3m$ is larger than the force from m and the net force is to the left. In each case the net force is away from equilibrium and the equilibrium is unstable.

(ii) If *M* is displaced a very small distance along the *y* axis in Figure 13.9, the net force is directed opposite to the direction of the displacement and therefore the equilibrium is stable.

EVALUATE: The point where the net force on *M* is zero is closer to the smaller mass.

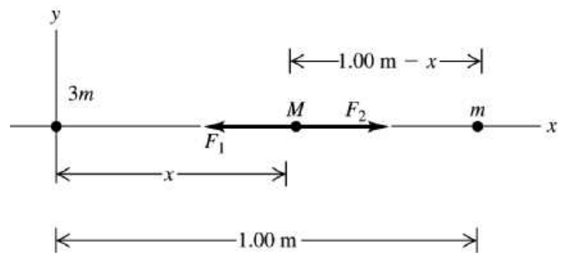


Figure 13.9

13.10. **IDENTIFY:** The force $\vec{F}_1$ exerted by m on M and the force $\vec{F}_2$ exerted by $2m$ on M are each given by Eq. (13.2) and the net force is the vector sum of these two forces.

SET UP: Each force is attractive. The forces on M in each region are sketched in Figure 13.10a. Let M be at coordinate x on the x-axis.

EXECUTE: **(a)** For the net force to be zero, $\vec{F}_1$ and $\vec{F}_2$ must be in opposite directions and this is the case only for $0 < x < L$. $\vec{F}_1 + \vec{F}_2 = 0$ then requires $F_1 = F_2$. $\dfrac{GmM}{x^2} = \dfrac{G(2m)M}{(L-x)^2}$. $2x^2 = (L-x)^2$ and $L - x = \pm\sqrt{2}x$. x must be less than L, so $x = \dfrac{L}{1+\sqrt{2}} = 0.414L$.

(b) For $x < 0$, $F_x > 0$. $F_x \to 0$ as $x \to -\infty$ and $F_x \to +\infty$ as $x \to 0$. For $x > L$, $F_x < 0$. $F_x \to 0$ as $x \to \infty$ and $F_x \to -\infty$ as $x \to L$. For $0 < x < 0.414L$, $F_x < 0$ and F_x increases from $-\infty$ to 0 as x goes from 0 to $0.414L$. For $0.414L < x < L$, $F_x > 0$ and F_x increases from 0 to $+\infty$ as x goes from $0.414L$ to L. The graph of F_x versus x is sketched in Figure 13.10b.

EVALUATE: Any real object is not exactly a point so it is not possible to have both m and M exactly at $x = 0$ or $2m$ and M both exactly at $x = L$. But the magnitude of the gravitational force between two objects approaches infinity as the objects get very close together.

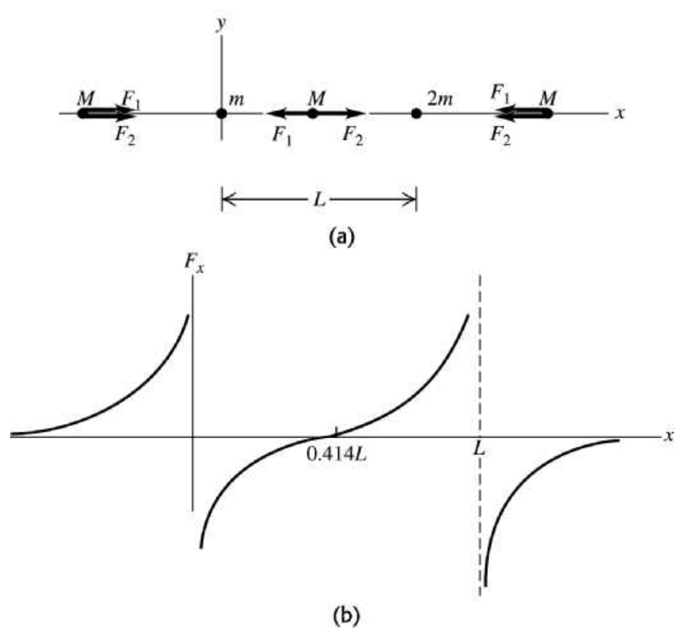


Figure 13.10

13.11. **IDENTIFY:** $F_g = G\frac{mm_E}{r^2}$, so $a_g = G\frac{m_E}{r^2}$, where r is the distance of the object from the center of the earth.

SET UP: $r = h + R_E$, where h is the distance of the object above the surface of the earth and $R_E = 6.38\times10^6$ m is the radius of the earth.

EXECUTE: To decrease the acceleration due to gravity by one-tenth, the distance from the center of the earth must be increased by a factor of $\sqrt{10}$, and so the distance above the surface of the earth is $(\sqrt{10}-1)R_E = 1.38\times10^7$ m.

EVALUATE: This height is about twice the radius of the earth.

13.12. **IDENTIFY:** Apply Eq. (13.4) to the earth and to Venus. $w = mg$.

SET UP: $g = \frac{Gm_E}{R_E^2} = 9.80\text{ m/s}^2$. $m_V = 0.815m_E$ and $R_V = 0.949R_E$. $w_E = mg_E = 75.0$ N.

EXECUTE: **(a)** $g_V = \frac{Gm_V}{R_V^2} = \frac{G(0.815m_E)}{(0.949R_E)^2} = 0.905\frac{Gm_E}{R_E^2} = 0.905g_E$.

(b) $w_V = mg_V = 0.905mg_E = (0.905)(75.0\text{ N}) = 67.9$ N.

EVALUATE: The mass of the rock is independent of its location but its weight equals the gravitational force on it and that depends on its location.

13.13. **(a) IDENTIFY** and **SET UP:** Apply Eq. (13.4) to the earth and to Titania. The acceleration due to gravity at the surface of Titania is given by $g_T = Gm_T/R_T^2$, where m_T is its mass and R_T is its radius.

For the earth, $g_E = Gm_E/R_E^2$.

EXECUTE: For Titania, $m_T = m_E/1700$ and $R_T = R_E/8$, so

$$g_T = \frac{Gm_T}{R_T^2} = \frac{G(m_E/1700)}{(R_E/8)^2} = \left(\frac{64}{1700}\right)\frac{Gm_E}{R_E^2} = 0.0377g_E.$$

Since $g_E = 9.80\text{ m/s}^2$, $g_T = (0.0377)(9.80\text{ m/s}^2) = 0.37\text{ m/s}^2$.

EVALUATE: g on Titania is much smaller than on earth. The smaller mass reduces g and is a greater effect than the smaller radius, which increases g.

(b) IDENTIFY and **SET UP:** Use density = mass/volume. Assume Titania is a sphere.

EXECUTE: From Section 13.2 we know that the average density of the earth is 5500 kg/m^3. For Titania

$$\rho_T = \frac{m_T}{\frac{4}{3}\pi R_T^3} = \frac{m_E/1700}{\frac{4}{3}\pi(R_E/8)^3} = \frac{512}{1700}\rho_E = \frac{512}{1700}(5500\text{ kg/m}^3) = 1700\text{ kg/m}^3.$$

EVALUATE: The average density of Titania is about a factor of 3 smaller than for earth. We can write Eq. (13.4) for Titania as $g_T = \frac{4}{3}\pi G R_T \rho_T$. $g_T < g_E$ both because $\rho_T < \rho_E$ and $R_T < R_E$.

13.14. **IDENTIFY:** Apply Eq. (13.4) to Rhea.

SET UP: $\rho = m/V$. The volume of a sphere is $V = \frac{4}{3}\pi R^3$.

EXECUTE: $M = \frac{gR^2}{G} = 2.44\times10^{21}$ kg and $\rho = \frac{M}{(4\pi/3)R^3} = 1.30\times10^3\text{ kg/m}^3$.

EVALUATE: The average density of Rhea is about one-fourth that of the earth.

13.15. **IDENTIFY:** Apply Eq. (13.2) to the astronaut.

SET UP: $m_E = 5.97\times10^{24}$ kg and $R_E = 6.38\times10^6$ m.

EXECUTE: $F_g = G\frac{mm_E}{r^2}$. $r = 600\times10^3\text{ m} + R_E$ so $F_g = 610$ N. At the surface of the earth, $w = mg = 735$ N. The gravity force is not zero in orbit. The satellite and the astronaut have the same acceleration so the astronaut's apparent weight is zero.

EVALUATE: In Eq. (13.2), r is the distance of the object from the center of the earth.

13.16. **IDENTIFY:** The gravity of Io limits the height to which volcanic material will rise. The acceleration due to gravity at the surface of Io depends on its mass and radius.

SET UP: The radius of Io is $R = 1.815\times10^6$ m. Use coordinates where $+y$ is upward. At the maximum height, $v_{0y} = 0$, $a_y = -g_{\text{Io}}$, which is assumed to be constant. Therefore the constant-acceleration kinematics formulas apply. The acceleration due to gravity at Io's surface is given by $g_{\text{Io}} = Gm/R^2$.

SOLVE: At the surface of Io, $g_{\text{Io}} = \dfrac{Gm}{R^2} = \dfrac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(8.94\times10^{22}\ \text{kg})}{(1.815\times10^6\ \text{m})^2} = 1.81\ \text{m/s}^2$. For constant acceleration (assumed), the equation $v_y^2 = v_{0y}^2 + 2a_y(y-y_0)$ applies, so $v_{0y} = \sqrt{-2a_y(y-y_0)} = \sqrt{-2(-1.81\ \text{m/s}^2)(5.00\times10^5\ \text{m})} = 1.345\times10^3$ m/s. Now solve for $y - y_0$ when $v_{0y} = 1.345\times10^3$ m/s and $a_y = -9.80\ \text{m/s}^2$. The equation $v_y^2 = v_{0y}^2 + 2a_y(y-y_0)$ gives

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{-(1.345\times10^3\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = 92\ \text{km}.$$

EVALUATE: Even though the mass of Io is around 100 times smaller than that of the earth, the acceleration due to gravity at its surface is only about 1/6 of that of the earth because Io's radius is much smaller than earth's radius.

13.17. **IDENTIFY:** The escape speed, from the results of Example 13.5, is $\sqrt{2GM/R}$.

SET UP: For Mars, $M = 6.42\times10^{23}$ kg and $R = 3.40\times10^6$ m. For Jupiter, $M = 1.90\times10^{27}$ kg and $R = 6.91\times10^7$ m.

EXECUTE: **(a)** $v = \sqrt{2(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(6.42\times10^{23}\ \text{kg})/(3.40\times10^6\ \text{m})} = 5.02\times10^3$ m/s.

(b) $v = \sqrt{2(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2(1.90\times10^{27}\ \text{kg})/(6.91\times10^7\ \text{m})} = 6.06\times10^4$ m/s.

(c) Both the kinetic energy and the gravitational potential energy are proportional to the mass of the spacecraft.

EVALUATE: Example 13.5 calculates the escape speed for earth to be 1.12×10^4 m/s. This is larger than our result for Mars and less than our result for Jupiter.

13.18. **IDENTIFY:** The kinetic energy is $K = \frac{1}{2}mv^2$ and the potential energy is $U = -\dfrac{GMm}{r}$.

SET UP: The mass of the earth is $M_{\text{E}} = 5.97\times10^{24}$ kg.

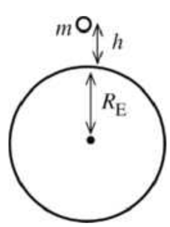
EXECUTE: **(a)** $K = \frac{1}{2}(629\ \text{kg})(3.33\times10^3\ \text{m/s})^2 = 3.49\times10^9$ J

(b) $U = -\dfrac{GM_{\text{E}}m}{r} = -\dfrac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})(629\ \text{kg})}{2.87\times10^9\ \text{m}} = -8.73\times10^7$ J.

EVALUATE: The total energy $K + U$ is positive.

13.19. **IDENTIFY:** Apply Newton's second law to the motion of the satellite and obtain an equation that relates the orbital speed v to the orbital radius r.

SET UP: The distances are shown in Figure 13.19a.



The radius of the orbit is $r = h + R_{\text{E}}$.

$r = 7.80\times10^5\ \text{m} + 6.38\times10^6\ \text{m} = 7.16\times10^6$ m.

Figure 13.19a

The free-body diagram for the satellite is given in Figure 13.19b.

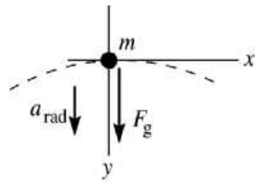


Figure 13.19b

(a) EXECUTE: $\Sigma F_y = ma_y$

$F_g = ma_{rad}$

$G\dfrac{mm_E}{r^2} = m\dfrac{v^2}{r}$

$$v = \sqrt{\frac{Gm_E}{r}} = \sqrt{\frac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}{7.16\times10^6\ \text{m}}} = 7.46\times10^3\ \text{m/s}$$

(b) $T = \dfrac{2\pi r}{v} = \dfrac{2\pi(7.16\times10^6\ \text{m})}{7.46\times10^3\ \text{m/s}} = 6030\ \text{s} = 1.68\ \text{h}.$

EVALUATE: Note that $r = h + R_E$ is the radius of the orbit, measured from the center of the earth. For this satellite r is greater than for the satellite in Example 13.6, so its orbital speed is less.

13.20. IDENTIFY: The time to complete one orbit is the period T, given by Eq. (13.12). The speed v of the satellite is given by $v = \dfrac{2\pi r}{T}$.

SET UP: If h is the height of the orbit above the earth's surface, the radius of the orbit is $r = h + R_E$. $R_E = 6.38\times10^6\ \text{m}$ and $m_E = 5.97\times10^{24}\ \text{kg}$.

EXECUTE: **(a)** $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_E}} = \dfrac{2\pi(7.05\times10^5\ \text{m} + 6.38\times10^6\ \text{m})^{3/2}}{\sqrt{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}} = 5.94\times10^3\ \text{s} = 99.0\ \text{min}$

(b) $v = \dfrac{2\pi(7.05\times10^5\ \text{m} + 6.38\times10^6\ \text{m})}{5.94\times10^3\ \text{s}} = 7.49\times10^3\ \text{m/s} = 7.49\ \text{km/s}$

EVALUATE: The satellite in Example 13.6 is at a lower altitude and therefore has a smaller orbit radius than the satellite in this problem. Therefore, the satellite in this problem has a larger period and a smaller orbital speed. But a large percentage change in h corresponds to a small percentage change in r and the values of T and v for the two satellites do not differ very much.

13.21. IDENTIFY: We know orbital data (speed and orbital radius) for one satellite and want to use it to find the orbital speed of another satellite having a known orbital radius. Newton's second law and the law of universal gravitation apply to both satellites.

SET UP: For circular motion, $F_{net} = ma = mv^2/r$, which in this case is $G\dfrac{mm_p}{r^2} = m\dfrac{v^2}{r}$.

EXECUTE: Using $G\dfrac{mm_p}{r^2} = m\dfrac{v^2}{r}$, we get $Gm_p = rv^2 = \text{constant}$. $r_1v_1^2 = r_2v_2^2$.

$$v_2 = v_1\sqrt{\frac{r_1}{r_2}} = (4800\ \text{m/s})\sqrt{\frac{5.00\times10^7\ \text{m}}{3.00\times10^7\ \text{m}}} = 6200\ \text{m/s}.$$

EVALUATE: The more distant satellite moves slower than the closer satellite, which is reasonable since the planet's gravity decreases with distance. The masses of the satellites do not affect their orbits.

13.22. IDENTIFY: We can calculate the orbital period T from the number of revolutions per day. Then the period and the orbit radius are related by Eq. (13.12).

SET UP: $m_E = 5.97\times10^{24}\ \text{kg}$ and $R_E = 6.38\times10^6\ \text{m}$. The height h of the orbit above the surface of the earth is related to the orbit radius r by $r = h + R_E$. $1\ \text{day} = 8.64\times10^4\ \text{s}$.

EXECUTE: The satellite moves 15.65 revolutions in 8.64×10^4 s, so the time for 1.00 revolution is

$T=\dfrac{8.64\times10^4\ \text{s}}{15.65}=5.52\times10^3\ \text{s}$. $T=\dfrac{2\pi r^{3/2}}{\sqrt{Gm_\text{E}}}$ gives

$r=\left(\dfrac{Gm_\text{E}T^2}{4\pi^2}\right)^{1/3}=\left(\dfrac{[6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2][5.97\times10^{24}\ \text{kg}][5.52\times10^3\ \text{s}]^2}{4\pi^2}\right)^{1/3}$. $r=6.75\times10^6$ m and

$h=r-R_\text{E}=3.7\times10^5\ \text{m}=370\ \text{km}$.

EVALUATE: The period of this satellite is slightly larger than the period for the satellite in Example 13.6 and the altitude of this satellite is therefore somewhat greater.

13.23. IDENTIFY: Apply $\Sigma\vec{F}=m\vec{a}$ to the motion of the baseball. $v=\dfrac{2\pi r}{T}$.

SET UP: $r_\text{D}=6\times10^3$ m.

EXECUTE: (a) $F_\text{g}=ma_\text{rad}$ gives $G\dfrac{m_\text{D}m}{r_\text{D}^2}=m\dfrac{v^2}{r_\text{D}}$.

$v=\sqrt{\dfrac{Gm_\text{D}}{r_\text{D}}}=\sqrt{\dfrac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(2.0\times10^{15}\ \text{kg})}{6\times10^3\ \text{m}}}=4.7\ \text{m/s}$

$4.7\ \text{m/s}=11\ \text{mph}$, which is easy to achieve.

(b) $T=\dfrac{2\pi r}{v}=\dfrac{2\pi(6\times10^3\ \text{m})}{4.7\ \text{m/s}}=8020\ \text{s}=134\ \text{min}=2.23\ \text{h}$. The game would last a long time.

EVALUATE: The speed v is relative to the center of Deimos. The baseball would already have some speed before we throw it, because of the rotational motion of Deimos.

13.24. IDENTIFY: $T=\dfrac{2\pi r}{v}$ and $F_\text{g}=ma_\text{rad}$.

SET UP: The sun has mass $m_\text{S}=1.99\times10^{30}$ kg. The radius of Mercury's orbit is 5.79×10^{10} m, so the radius of Vulcan's orbit is 3.86×10^{10} m.

EXECUTE: $F_\text{g}=ma_\text{rad}$ gives $G\dfrac{m_\text{S}m}{r^2}=m\dfrac{v^2}{r}$ and $v^2=\dfrac{Gm_\text{S}}{r}$.

$T=2\pi r\sqrt{\dfrac{r}{Gm_\text{S}}}=\dfrac{2\pi r^{3/2}}{\sqrt{Gm_\text{S}}}=\dfrac{2\pi(3.86\times10^{10}\ \text{m})^{3/2}}{\sqrt{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})}}=4.13\times10^6\ \text{s}=47.8\ \text{days}$

EVALUATE: The orbital period of Mercury is 88.0 d, so we could calculate T for Vulcan as $T=(88.0\ \text{d})(2/3)^{3/2}=47.9$ days.

13.25. IDENTIFY: The orbital speed is given by $v=\sqrt{Gm/r}$, where m is the mass of the star. The orbital period is given by $T=\dfrac{2\pi r}{v}$.

SET UP: The sun has mass $m_\text{S}=1.99\times10^{30}$ kg. The orbit radius of the earth is 1.50×10^{11} m.

EXECUTE: (a) $v=\sqrt{Gm/r}$.

$v=\sqrt{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(0.85\times1.99\times10^{30}\ \text{kg})/((1.50\times10^{11}\ \text{m})(0.11))}=8.27\times10^4\ \text{m/s}$.

(b) $2\pi r/v=1.25\times10^6\ \text{s}=14.5$ days (about two weeks).

EVALUATE: The orbital period is less than the 88-day orbital period of Mercury; this planet is orbiting very close to its star, compared to the orbital radius of Mercury.

13.26. IDENTIFY: The period of each satellite is given by Eq. (13.12). Set up a ratio involving T and r.

SET UP: $T=\dfrac{2\pi r^{3/2}}{\sqrt{Gm_\text{p}}}$ gives $\dfrac{T}{r^{3/2}}=\dfrac{2\pi}{\sqrt{Gm_\text{p}}}=\text{constant}$, so $\dfrac{T_1}{r_1^{3/2}}=\dfrac{T_2}{r_2^{3/2}}$.

EXECUTE: $T_2 = T_1\left(\dfrac{r_2}{r_1}\right)^{3/2} = (6.39 \text{ days})\left(\dfrac{48{,}000 \text{ km}}{19{,}600 \text{ km}}\right)^{3/2} = 24.5$ days. For the other satellite,

$$T_2 = (6.39 \text{ days})\left(\frac{64{,}000 \text{ km}}{19{,}600 \text{ km}}\right)^{3/2} = 37.7 \text{ days}.$$

EVALUATE: T increases when r increases.

13.27. **IDENTIFY:** In part (b) apply the results from part (a).

SET UP: For Pluto, $e = 0.248$ and $a = 5.92\times10^{12}$ m. For Neptune, $e = 0.010$ and $a = 4.50\times10^{12}$ m. The orbital period for Pluto is $T = 247.9$ y.

EXECUTE: **(a)** The result follows directly from Figure 13.18 in the textbook.

(b) The closest distance for Pluto is $(1-0.248)(5.92\times10^{12}\text{ m}) = 4.45\times10^{12}$ m. The greatest distance for Neptune is $(1+0.010)(4.50\times10^{12}\text{ m}) = 4.55\times10^{12}$ m.

(c) The time is the orbital period of Pluto, $T = 248$ y.

EVALUATE: Pluto's closest distance calculated in part (a) is $0.10\times10^{12}\text{ m} = 1.0\times10^{8}\text{ km},$ so Pluto is about 100 million km closer to the sun than Neptune, as is stated in the problem. The eccentricity of Neptune's orbit is small, so its distance from the sun is approximately constant.

13.28. **IDENTIFY:** $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_{\text{star}}}}$, where m_{star} is the mass of the star. $v = \dfrac{2\pi r}{T}$.

SET UP: $3.09 \text{ days} = 2.67\times10^{5}$ s. The orbit radius of Mercury is 5.79×10^{10} m. The mass of our sun is 1.99×10^{30} kg.

EXECUTE: **(a)** $T = 2.67\times10^{5}$ s. $r = (5.79\times10^{10}\text{ m})/9 = 6.43\times10^{9}$ m. $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_{\text{star}}}}$ gives

$$m_{\text{star}} = \frac{4\pi^2 r^3}{T^2 G} = \frac{4\pi^2(6.43\times10^{9}\text{ m})^3}{(2.67\times10^{5}\text{ s})^2(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)} = 2.21\times10^{30}\text{ kg}.\quad \frac{m_{\text{star}}}{m_{\text{sun}}} = 1.11, \text{ so}$$

$m_{\text{star}} = 1.11 m_{\text{sun}}$.

(b) $v = \dfrac{2\pi r}{T} = \dfrac{2\pi(6.43\times10^{9}\text{ m})}{2.67\times10^{5}\text{ s}} = 1.51\times10^{5}\text{ m/s} = 151\text{ km/s}$

EVALUATE: The orbital period of Mercury is 88.0 d. The period for this planet is much less primarily because the orbit radius is much less and also because the mass of the star is greater than the mass of our sun.

13.29. **IDENTIFY:** Knowing the orbital radius and orbital period of a satellite, we can calculate the mass of the object about which it is revolving.

SET UP: The radius of the orbit is $r = 10.5\times10^{9}$ m and its period is $T = 6.3 \text{ days} = 5.443\times10^{5}$ s. The mass of the sun is $m_S = 1.99\times10^{30}$ kg. The orbital period is given by $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_{\text{HD}}}}$.

EXECUTE: Solving $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_{\text{HD}}}}$ for the mass of the star gives

$$m_{\text{HD}} = \frac{4\pi^2 r^3}{T^2 G} = \frac{4\pi^2(10.5\times10^{9}\text{ m})^3}{(5.443\times10^{5}\text{ s})^2(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)} = 2.3\times10^{30}\text{ kg}, \text{ which is } m_{\text{HD}} = 1.2 m_S.$$

EVALUATE: The mass of the star is only 20% greater than that of our sun, yet the orbital period of the planet is much shorter than that of the earth, so the planet must be much closer to the star than the earth is.

13.30. **IDENTIFY:** Section 13.6 states that for a point mass outside a spherical shell the gravitational force is the same as if all the mass of the shell were concentrated at its center. It also states that for a point inside a spherical shell the force is zero.

SET UP: For $r = 5.01$ m the point mass is outside the shell and for $r = 4.99$ m and $r = 2.72$ m the point mass is inside the shell.

EXECUTE: **(a)** (i) $F_g = \dfrac{Gm_1m_2}{r^2} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(1000.0\ \text{kg})(2.00\ \text{kg})}{(5.01\ \text{m})^2} = 5.31\times10^{-9}\ \text{N}.$

(ii) $F_g = 0.$ (iii) $F_g = 0.$

(b) For $r < 5.00$ m the force is zero and for $r > 5.00$ m the force is proportional to $1/r^2$. The graph of F_g versus r is sketched in Figure 13.30.

EVALUATE: Inside the shell the gravitational potential energy is constant and the force on a point mass inside the shell is zero.

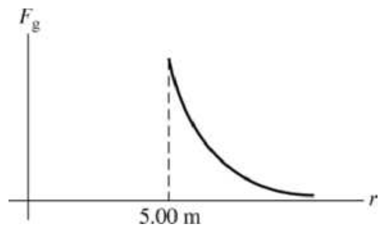


Figure 13.30

13.31. **IDENTIFY:** Section 13.6 states that for a point mass outside a uniform sphere the gravitational force is the same as if all the mass of the sphere were concentrated at its center. It also states that for a point mass a distance r from the center of a uniform sphere, where r is less than the radius of the sphere, the gravitational force on the point mass is the same as though we removed all the mass at points farther than r from the center and concentrated all the remaining mass at the center.

SET UP: The density of the sphere is $\rho = \dfrac{M}{\frac{4}{3}\pi R^3}$, where M is the mass of the sphere and R is its radius.

The mass inside a volume of radius $r < R$ is $M_r = \rho V_r = \left(\dfrac{M}{\frac{4}{3}\pi R^3}\right)\left(\tfrac{4}{3}\pi r^3\right) = M\left(\dfrac{r}{R}\right)^3$. $r = 5.01$ m is outside the sphere and $r = 2.50$ m is inside the sphere.

EXECUTE: **(a)** (i) $F_g = \dfrac{GMm}{r^2} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(1000.0\ \text{kg})(2.00\ \text{kg})}{(5.01\ \text{m})^2} = 5.31\times10^{-9}\ \text{N}.$

(ii) $F_g = \dfrac{GM'm}{r^2}$. $M' = M\left(\dfrac{r}{R}\right)^3 = (1000.0\ \text{kg})\left(\dfrac{2.50\ \text{m}}{5.00\ \text{m}}\right)^3 = 125\ \text{kg}.$

$$F_g = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(125\ \text{kg})(2.00\ \text{kg})}{(2.50\ \text{m})^2} = 2.67\times10^{-9}\ \text{N}.$$

(b) $F_g = \dfrac{GM(r/R)^3 m}{r^2} = \left(\dfrac{GMm}{R^3}\right)r$ for $r < R$ and $F_g = \dfrac{GMm}{r^2}$ for $r > R$. The graph of F_g versus r is sketched in Figure 13.31.

EVALUATE: At points outside the sphere the force on a point mass is the same as for a shell of the same mass and radius. For $r < R$ the force is different in the two cases of uniform sphere versus hollow shell.

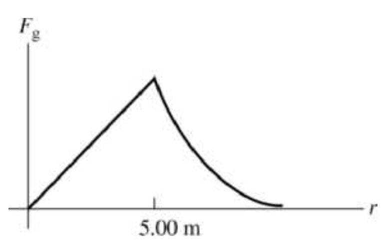


Figure 13.31

13.32. **IDENTIFY:** The gravitational potential energy of a pair of point masses is $U = -G\dfrac{m_1 m_2}{r}$. Divide the rod into infinitesimal pieces and integrate to find U.

SET UP: Divide the rod into differential masses dm at position l, measured from the right end of the rod. $dm = dl(M/L)$.

EXECUTE: **(a)** $U = -\dfrac{Gm\,dm}{l+x} = -\dfrac{GmM}{L}\dfrac{dl}{l+x}$.

Integrating, $U - \dfrac{GmM}{L}\displaystyle\int_0^L \frac{dl}{l+x} = -\frac{GmM}{L}\ln\left(1+\frac{L}{x}\right)$. For $x \gg L$, the natural logarithm is $\sim(L/x)$, and $U \to -GmM/x$.

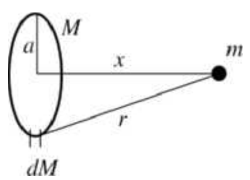
(b) The x-component of the gravitational force on the sphere is

$F_x = -\dfrac{\partial U}{\partial x} = \dfrac{GmM}{L}\dfrac{(-L/x^2)}{(1+(L/x))} = -\dfrac{GmM}{(x^2+Lx)}$, with the minus sign indicating an attractive force. As $x \gg L$, the denominator in the above expression approaches x^2, and $F_x \to -GmM/x^2$, as expected.

EVALUATE: When x is much larger than L the rod can be treated as a point mass, and our results for U and F_x do reduce to the correct expression when $x \gg L$.

13.33. **IDENTIFY:** Find the potential due to a small segment of the ring and integrate over the entire ring to find the total U.

(a) SET UP:



Divide the ring up into small segments dM, as indicated in Figure 13.33.

Figure 13.33

EXECUTE: The gravitational potential energy of dM and m is $dU = -GmdM/r$.

The total gravitational potential energy of the ring and particle is $U = \int dU = -Gm\int dM/r$.

But $r = \sqrt{x^2+a^2}$ is the same for all segments of the ring, so

$$U = -\frac{Gm}{r}\int dM = -\frac{GmM}{r} = -\frac{GmM}{\sqrt{x^2+a^2}}.$$

(b) EVALUATE: When $x \gg a$, $\sqrt{x^2+a^2} \to \sqrt{x^2} = x$ and $U = -GmM/x$. This is the gravitational potential energy of two point masses separated by a distance x. This is the expected result.

(c) IDENTIFY and **SET UP:** Use $F_x = -dU/dx$ with $U(x)$ from part (a) to calculate F_x.

EXECUTE: $F_x = -\dfrac{dU}{dx} = -\dfrac{d}{dx}\left(-\dfrac{GmM}{\sqrt{x^2+a^2}}\right)$

$$F_x = +GmM\frac{d}{dx}(x^2+a^2)^{-1/2} = GmM\left(-\frac{1}{2}(2x)(x^2+a^2)^{-3/2}\right)$$

$F_x = -GmMx/(x^2+a^2)^{3/2}$; the minus sign means the force is attractive.

EVALUATE: **(d)** For $x \gg a$, $(x^2+a^2)^{3/2} \to (x^2)^{3/2} = x^3$

Then $F_x = -GmMx/x^3 = -GmM/x^2$. This is the force between two point masses separated by a distance x and is the expected result.

(e) For $x = 0$, $U = -GMm/a$. Each small segment of the ring is the same distance from the center and the potential is the same as that due to a point charge of mass M located at a distance a.

For $x=0$, $F_x=0$. When the particle is at the center of the ring, symmetrically placed segments of the ring exert equal and opposite forces and the total force exerted by the ring is zero.

13.34. **IDENTIFY:** At the north pole, Sneezy has no circular motion and therefore no acceleration. But at the equator he has acceleration toward the center of the earth due to the earth's rotation.

SET UP: The earth has mass $m_E = 5.97\times10^{24}$ kg, radius $R_E = 6.38\times10^6$ m and rotational period $T = 24\text{ hr} = 8.64\times10^4$ s. Use coordinates for which the $+y$ direction is toward the center of the earth. The free-body diagram for Sneezy at the equator is given in Figure 13.34. The radial acceleration due to Sneezy's circular motion at the equator is $a_{rad} = \dfrac{4\pi^2 R}{T^2}$, and Newton's second law applies to Sneezy.

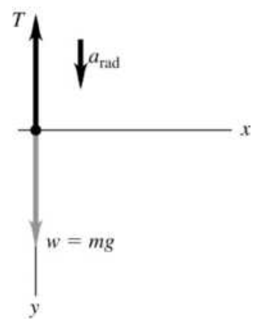


Figure 13.34

EXECUTE: At the north pole Sneezy has $a=0$ and $T = w = 475.0$ N (the gravitational force exerted by the earth). Sneezy has mass $w/g = 48.47$ kg. At the equator Sneezy is traveling in a circular path and has radial acceleration $a_{rad} = \dfrac{4\pi^2 R}{T^2} = \dfrac{4\pi^2(6.38\times10^6\text{ m})}{(8.64\times10^4\text{ s})^2} = 0.0337\text{ m/s}^2$. Newton's second law $\Sigma F_y = ma_y$ gives $w - T = ma_{rad}$. Solving for T gives

$T = w - ma_{rad} = m(g - a_{rad}) = (48.47\text{ kg})(9.80\text{ m/s}^2 - 0.0337\text{ m/s}^2) = 473.4\text{ N}.$

EVALUATE: At the equator Sneezy has an inward acceleration and the outward tension is less than the true weight, since there is a net inward force.

13.35. **IDENTIFY** and **SET UP:** At the north pole, $F_g = w_0 = mg_0$, where g_0 is given by Eq. (13.4) applied to Neptune. At the equator, the apparent weight is given by Eq. (13.28). The orbital speed v is obtained from the rotational period using Eq. (13.12).

EXECUTE: **(a)** $g_0 = Gm/R^2 = (6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.0\times10^{26}\text{ kg})/(2.5\times10^7\text{ m})^2 = 10.7\text{ m/s}^2$. This agrees with the value of g given in the problem.

$F = w_0 = mg_0 = (5.0\text{ kg})(10.7\text{ m/s}^2) = 53$ N; this is the true weight of the object.

(b) From Eq. (13.28), $w = w_0 - mv^2/R$

$T = \dfrac{2\pi r}{v}$ gives $v = \dfrac{2\pi r}{T} = \dfrac{2\pi(2.5\times10^7\text{ m})}{(16\text{ h})(3600\text{ s/1 h})} = 2.727\times10^3$ m/s

$v^2/R = (2.727\times10^3\text{ m/s})^2/2.5\times10^7\text{ m} = 0.297\text{ m/s}^2$

Then $w = 53\text{ N} - (5.0\text{ kg})(0.297\text{ m/s}^2) = 52$ N.

EVALUATE: The apparent weight is less than the true weight. This effect is larger on Neptune than on earth.

13.36. **IDENTIFY:** The radius of a black hole and its mass are related by $R_S = \dfrac{2GM}{c^2}$.

SET UP: $R_S = 0.50\times10^{-15}$ m, $G = 6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2$ and $c = 3.00\times10^8$ m/s.

EXECUTE: $M = \dfrac{c^2 R_S}{2G} = \dfrac{(3.00\times10^8 \text{ m/s})^2(0.50\times10^{-15} \text{ m})}{2(6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)} = 3.4\times10^{11} \text{ kg}$

EVALUATE: The average density of the black hole would be

$\rho = \dfrac{M}{\frac{4}{3}\pi R_S^3} = \dfrac{3.4\times10^{11} \text{ kg}}{\frac{4}{3}\pi(0.50\times10^{-15} \text{ m})^3} = 6.49\times10^{56} \text{ kg/m}^3$. We can combine $\rho = \dfrac{M}{\frac{4}{3}\pi R_S^3}$ and $R_S = \dfrac{2GM}{c^2}$ to give $\rho = \dfrac{3c^6}{32\pi G^3 M^2}$. The average density of a black hole increases when its mass decreases. The average density of this mini black hole is much greater than the average density of the much more massive black hole in Example 13.11.

13.37. **IDENTIFY:** The orbital speed for an object a distance r from an object of mass M is $v = \sqrt{\dfrac{GM}{r}}$. The mass M of a black hole and its Schwarzschild radius R_S are related by Eq. (13.30).

SET UP: $c = 3.00\times10^8$ m/s. $1 \text{ ly} = 9.461\times10^{15}$ m.

EXECUTE: **(a)**

$$M = \frac{rv^2}{G} = \frac{(7.5 \text{ ly})(9.461\times10^{15} \text{ m/ly})(200\times10^3 \text{ m/s})^2}{(6.673\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)} = 4.3\times10^{37} \text{ kg} = 2.1\times10^7 \text{ M}_S.$$

(b) No, the object has a mass very much greater than 50 solar masses.

(c) $R_S = \dfrac{2GM}{c^2} = \dfrac{2v^2 r}{c^2} = 6.32\times10^{10}$ m, which does fit.

EVALUATE: The Schwarzschild radius of a black hole is approximately the same as the radius of Mercury's orbit around the sun.

13.38. **IDENTIFY:** Apply Eq. (13.1) to calculate the gravitational force. For a black hole, the mass M and Schwarzschild radius R_S are related by Eq. (13.30).

SET UP: The speed of light is $c = 3.00\times10^8$ m/s.

EXECUTE: **(a)** $\dfrac{GMm}{r^2} = \dfrac{(R_S c^2/2)m}{r^2} = \dfrac{mc^2 R_S}{2r^2}$.

(b) $\dfrac{(5.00 \text{ kg})(3.00\times10^8 \text{ m/s})^2(1.4\times10^{-2} \text{ m})}{2(3.00\times10^6 \text{ m})^2} = 350 \text{ N}$.

(c) Solving Eq. (13.30) for M, $M = \dfrac{R_S c^2}{2G} = \dfrac{(14.00\times10^{-3} \text{ m})\,(3.00\times10^8 \text{ m/s})^2}{2(6.673\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)} = 9.44\times10^{24} \text{ kg}$.

EVALUATE: The mass of the black hole is about twice the mass of the earth.

13.39. **IDENTIFY:** The clumps orbit the black hole. Their speed, orbit radius and orbital period are related by $v = \dfrac{2\pi r}{T}$. Their orbit radius and period are related to the mass M of the black hole by $T = \dfrac{2\pi r^{3/2}}{\sqrt{GM}}$. The radius of the black hole's event horizon is related to the mass of the black hole by $R_S = \dfrac{2GM}{c^2}$.

SET UP: $v = 3.00\times10^7$ m/s. $T = 27 \text{ h} = 9.72\times10^4$ s. $c = 3.00\times10^8$ m/s.

EXECUTE: **(a)** $r = \dfrac{vT}{2\pi} = \dfrac{(3.00\times10^7 \text{ m/s})(9.72\times10^4 \text{ s})}{2\pi} = 4.64\times10^{11} \text{ m}$.

(b) $T = \dfrac{2\pi r^{3/2}}{\sqrt{GM}}$ gives $M = \dfrac{4\pi^2 r^3}{GT^2} = \dfrac{4\pi^2(4.64\times10^{11} \text{ m})^3}{(6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(9.72\times10^4 \text{ s})^2} = 6.26\times10^{36} \text{ kg}$.

$= 3.15\times10^6 M_S$, where M_S is the mass of our sun

(c) $R_S = \dfrac{2GM}{c^2} = \dfrac{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(6.26\times10^{36}\ \text{kg})}{(3.00\times10^8\ \text{m/s})^2} = 9.28\times10^9\ \text{m}$

EVALUATE: The black hole has a mass that is about 3×10^6 solar masses.

13.40. **IDENTIFY:** Apply Eq. (13.1) to calculate the magnitude of each gravitational force. Each force is attractive.

SET UP: The forces on one of the masses are sketched in Figure 13.40. The figure shows that the vector sum of the three forces is toward the center of the square.

EXECUTE: $F_{\text{onA}} = 2F_B\cos 45° + F_D = 2\dfrac{Gm_Am_B\cos 45°}{r_{AB}^2} + \dfrac{Gm_Am_D}{r_{AD}^2}$.

$$F_{\text{onA}} = \frac{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(800\ \text{kg})^2\cos 45°}{(0.10\ \text{m})^2} + \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(800\ \text{kg})^2}{2(0.10\ \text{m})^2} = 8.2\times10^{-3}\ \text{N}$$

toward the center of the square.

EVALUATE: We have assumed each mass can be treated as a uniform sphere. Each mass must have an unusually large density in order to have mass 800 kg and still fit into a square of side length 10.0 cm.

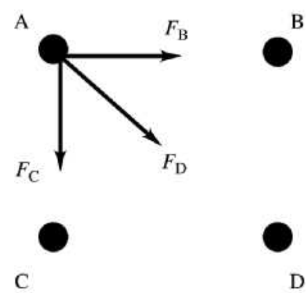


Figure 13.40

13.41. **IDENTIFY:** $g_n = G\dfrac{m_n}{R_n^2}$, where the subscript n refers to the neutron star. $w = mg$.

SET UP: $R_n = 10.0\times10^3$ m. $m_n = 1.99\times10^{30}$ kg. Your mass is $m = \dfrac{w}{g} = \dfrac{675\ \text{N}}{9.80\ \text{m/s}^2} = 68.9$ kg.

EXECUTE: $g_n = (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\dfrac{1.99\times10^{30}\ \text{kg}}{(10.0\times10^3\ \text{m})^2} = 1.33\times10^{12}\ \text{m/s}^2$

Your weight on the neutron star would be $w_n = mg_n = (68.9\ \text{kg})(1.33\times10^{12}\ \text{m/s}^2) = 9.16\times10^{13}$ N.

EVALUATE: Since R_n is much less than the radius of the sun, the gravitational force exerted by the neutron star on an object at its surface is immense.

13.42. **IDENTIFY:** Use Eq. (13.4) to calculate g for Europa. The acceleration of a particle moving in a circular path is $a_{\text{rad}} = r\omega^2$.

SET UP: In $a_{\text{rad}} = r\omega^2$, ω must be in rad/s. For Europa, $R = 1.569\times10^6$ m.

EXECUTE: $g = \dfrac{Gm}{R^2} = \dfrac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(4.8\times10^{22}\ \text{kg})}{(1.569\times10^6\ \text{m})^2} = 1.30\ \text{m/s}^2$. $g = a_{\text{rad}}$ gives

$$\omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{1.30\ \text{m/s}^2}{4.25\ \text{m}}} = (0.553\ \text{rad/s})\left(\frac{60\ \text{s}}{1\ \text{min}}\right)\left(\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right) = 5.28\ \text{rpm}.$$

EVALUATE: The radius of Europa is about one-fourth that of the earth and its mass is about one-hundredth that of earth, so g on Europa is much less than g on earth. The lander would have some spatial extent so different points on it would be different distances from the rotation axis and a_{rad} would have different values. For the ω we calculated, $a_{\text{rad}} = g$ at a point that is precisely 4.25 m from the rotation axis.

13.43. **IDENTIFY:** Use Eq. (13.1) to find each gravitational force. Each force is attractive. In part (b) apply conservation of energy.

SET UP: For a pair of masses m_1 and m_2 with separation r, $U = -G\dfrac{m_1 m_2}{r}$.

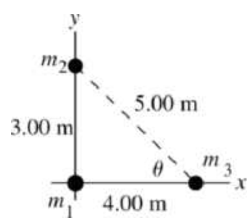
EXECUTE: **(a)** From symmetry, the net gravitational force will be in the direction $45°$ from the x-axis (bisecting the x and y axes), with magnitude

$$F = (6.673\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(0.0150\text{ kg})\left[\frac{(2.0\text{ kg})}{(2(0.50\text{ m})^2)} + 2\frac{(1.0\text{ kg})}{(0.50\text{ m})^2}\sin 45°\right] = 9.67\times10^{-12}\text{ N}$$

(b) The initial displacement is so large that the initial potential energy may be taken to be zero. From the work-energy theorem, $\dfrac{1}{2}mv^2 = Gm\left[\dfrac{(2.0\text{ kg})}{\sqrt{2}\,(0.50\text{ m})} + 2\dfrac{(1.0\text{ kg})}{(0.50\text{ m})}\right]$. Canceling the factor of m and solving for v, and using the numerical values gives $v = 3.02\times10^{-5}$ m/s.

EVALUATE: The result in part (b) is independent of the mass of the particle. It would take the particle a long time to reach point P.

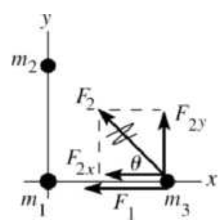
13.44. **IDENTIFY:** Use Eq. (13.1) to calculate each gravitational force and add the forces as vectors.
(a) SET UP: The locations of the masses are sketched in Figure 13.44a.



Section 13.6 proves that any two spherically symmetric masses interact as though they were point masses with all the mass concentrated at their centers.

Figure 13.44a

The force diagram for m_3 is given in Figure 13.44b.



$\cos\theta = 0.800$
$\sin\theta = 0.600$

Figure 13.44b

EXECUTE: $F_1 = G\dfrac{m_1 m_3}{r_{13}^2} = \dfrac{(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(60.0\text{ kg})(0.500\text{ kg})}{(4.00\text{ m})^2} = 1.251\times10^{-10}\text{ N}$

$$F_2 = G\frac{m_2 m_3}{r_{23}^2} = \frac{(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(80.0\text{ kg})(0.500\text{ kg})}{(5.00\text{ m})^2} = 1.068\times10^{-10}\text{ N}$$

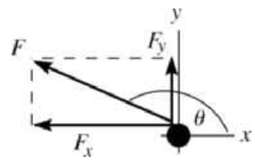
$F_{1x} = -1.251\times10^{-10}$ N, $F_{1y} = 0$

$F_{2x} = -F_2\cos\theta = -(1.068\times10^{-10}\text{ N})(0.800) = -8.544\times10^{-11}\text{ N}$

$F_{2y} = +F_2\sin\theta = +(1.068\times10^{-10}\text{ N})(0.600) = +6.408\times10^{-11}\text{ N}$

$F_x = F_{1x} + F_{2x} = -1.251\times10^{-10}\text{ N} - 8.544\times10^{-11}\text{ N} = -2.105\times10^{-10}\text{ N}$

$F_y = F_{1y} + F_{2y} = 0 + 6.408\times10^{-11}\text{ N} = +6.408\times10^{-11}\text{ N}$



F and its components are sketched in Figure 13.44c.

$$F=\sqrt{F_x^2+F_y^2}$$

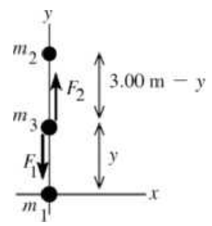
$$F=\sqrt{(-2.105\times10^{-10}\text{ N})^2+(+6.408\times10^{-11}\text{ N})^2}$$

$$F=2.20\times10^{-10}\text{ N}$$

$$\tan\theta=\frac{F_y}{F_x}=\frac{+6.408\times10^{-11}\text{ N}}{-2.105\times10^{-10}\text{ N}};\ \ \theta=163°$$

Figure 13.44c

EVALUATE: Both spheres attract the third sphere and the net force is in the second quadrant.
(b) SET UP: For the net force to be zero the forces from the two spheres must be equal in magnitude and opposite in direction. For the forces on it to be opposite in direction the third sphere must be on the y-axis and between the other two spheres. The forces on the third sphere are shown in Figure 13.44d.



EXECUTE: $F_{\text{net}}=0$ if $F_1=F_2$

$$G\frac{m_1m_3}{y^2}=G\frac{m_2m_3}{(3.00\text{ m}-y)^2}$$

$$\frac{60.0}{y^2}=\frac{80.0}{(3.00\text{ m}-y)^2}$$

Figure 13.44d

$$\sqrt{80.0}y=\sqrt{60.0}(3.00\text{ m}-y)$$

$$(\sqrt{80.0}+\sqrt{60.0})y=(3.00\text{ m})\sqrt{60.0}\ \text{ and }\ y=1.39\text{ m}$$

Thus the sphere would have to be placed at the point $x=0,\ \ y=1.39$ m.

EVALUATE: For the forces to have the same magnitude the third sphere must be closer to the sphere that has smaller mass.

13.45. IDENTIFY: The mass and radius of the moon determine the acceleration due to gravity at its surface. This in turn determines the normal force on the hip, which then determines the kinetic friction force while walking.

SET UP: $m_{\text{M}}=7.35\times10^{22}$ kg, $R_{\text{M}}=1.74\times10^{6}$ m. The mass supported by the hip is $(0.65)(65\text{ kg})+43\text{ kg}=85.25$ kg. The acceleration due to gravity on the moon is $g_{\text{M}}=\dfrac{Gm_{\text{M}}}{{R_{\text{M}}}^2}$ and $f_{\text{k}}=\mu_{\text{k}}n.$

EXECUTE: **(a)** The acceleration due to gravity on the moon is

$$g_{\text{M}}=\frac{Gm_{\text{M}}}{{R_{\text{M}}}^2}=\frac{(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(7.35\times10^{22}\text{ kg})}{(1.74\times10^{6}\text{ m})^2}=1.62\text{ m/s}^2.$$

(b) $n=(85.25\text{ kg})g_{\text{M}}=138$ N and $f_{\text{k}}=\mu_{\text{k}}n=(0.0050)(138\text{ N})=0.69$ N.

(c) $n=(85.25\text{ kg})g_{\text{E}}=835$ N and $f_{\text{k}}=\mu_{\text{k}}n=4.2$ N.

EVALUATE: Walking on the moon should produce much less wear on the hip joints than on the earth.

13.46. IDENTIFY: The gravitational pulls of Titan and Saturn on the *Huygens* probe should be in opposite directions and of equal magnitudes to cancel.

SET UP: The mass of Saturn is $m_{\text{S}}=5.68\times10^{26}$ kg. When the probe is a distance d from the center of Titan it is a distance $1.22\times10^{9}\text{ m}-d$ from the center of Saturn. The magnitude of the gravitational force is given by $F_{\text{grav}}=GmM/r^2$.

EXECUTE: Equal gravity forces means the two gravitational pulls on the probe must balance, so $G\frac{mm_T}{d^2} = G\frac{mm_S}{(1.22\times10^9\text{ m}-d)^2}$. Simplifying, this becomes $d = \sqrt{\frac{m_T}{m_S}}(1.22\times10^9\text{ m}-d)$. Using the masses from the text and solving for d we get $d = \sqrt{\frac{1.35\times10^{23}\text{ kg}}{5.68\times10^{26}\text{ kg}}}(1.22\times10^9\text{ m}-d) = (0.0154)(1.22\times10^9\text{ m}-d)$, so $d = 1.85\times10^7\text{ m} = 1.85\times10^4\text{ km}$.

EVALUATE: For the forces to balance, the probe must be much closer to Titan than to Saturn since Titan's mass is much smaller than that of Saturn.

13.47. IDENTIFY: Knowing the density and radius of Toro, we can calculate its mass and then the acceleration due to gravity at its surface. We can then use orbital mechanics to determine its orbital speed knowing the radius of its orbit.

SET UP: Density is $\rho = m/V$, and the volume of a sphere is $\frac{4}{3}\pi R^3$. Use the assumption that the density of Toro is the same as that of earth to calculate the mass of Toro. Then $g_T = G\frac{m_T}{R_T^2}$. Apply $\Sigma\vec{F} = m\vec{a}$ to the object to find its speed when it is in a circular orbit around Toro.

EXECUTE: **(a)** Toro and the earth are assumed to have the same densities, so $\frac{m_E}{\frac{4}{3}\pi R_E^3} = \frac{m_T}{\frac{4}{3}\pi R_T^3}$ gives

$$m_T = m_E\left(\frac{R_T}{R_E}\right)^3 = (5.97\times10^{24}\text{ kg})\left(\frac{5.0\times10^3\text{ m}}{6.38\times10^6\text{ m}}\right)^3 = 2.9\times10^{15}\text{ kg}.$$

$$g_T = G\frac{m_T}{R_T^2} = \frac{(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(2.9\times10^{15}\text{ kg})}{(5.0\times10^3\text{ m})^2} = 7.7\times10^{-3}\text{ m/s}^2.$$

(b) The force of gravity on the object is mg_T. In a circular orbit just above the surface of Toro, its acceleration is $\frac{v^2}{R_T}$. Then $\Sigma\vec{F} = m\vec{a}$ gives $mg_T = m\frac{v^2}{R_T}$ and

$$v = \sqrt{g_T R_T} = \sqrt{(7.7\times10^{-3}\text{ m/s}^2)(5.0\times10^3\text{ m})} = 6.2\text{ m/s}.$$

EVALUATE: A speed of 6.2 m/s corresponds to running 100 m in 16.1 s, which is barely possible for the average person, but a well-conditioned athlete might do it.

13.48. IDENTIFY: The gravity force for each pair of objects is given by Eq. (13.1). The work done is $W = -\Delta U$.

SET UP: The simplest way to approach this problem is to find the force between the spacecraft and the center of mass of the earth-moon system, which is $4.67\times10^6\text{ m}$ from the center of the earth. The distance from the spacecraft to the center of mass of the earth-moon system is $3.82\times10^8\text{ m}$ (Figure 13.48).

$m_E = 5.97\times10^{24}\text{ kg}$, $m_M = 7.35\times10^{22}\text{ kg}$.

EXECUTE: **(a)** Using the Law of Gravitation, the force on the spacecraft is 3.4 N, an angle of $0.61°$ from the earth-spacecraft line.

(b) $U = -G\frac{m_A m_B}{r}$. $U_2 = 0$ and $r_1 = 3.84\times10^8\text{ m}$ for the spacecraft and the earth, and the spacecraft and the moon.

$$W = U_2 - U_1 = +\frac{GMm}{r_1} = +\frac{(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg} + 7.35\times10^{22}\text{ kg})(1250\text{ kg})}{3.84\times10^8\text{ m}}.$$

$W = 1.31\times10^9\text{ J}.$

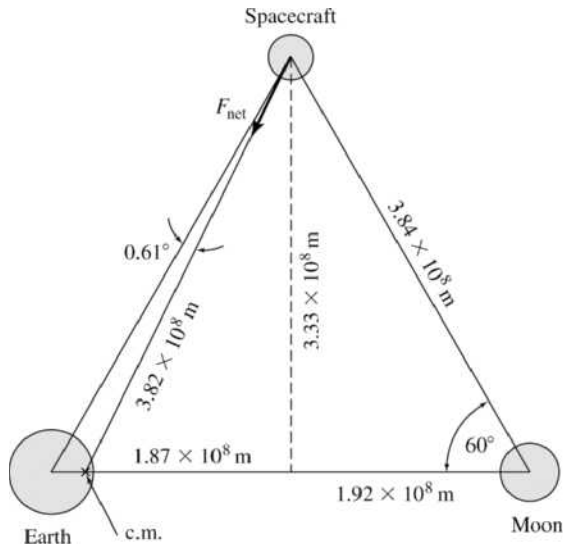


Figure 13.48

EVALUATE: The work done by the attractive gravity forces is negative. The work you do is positive.

13.49. **IDENTIFY:** Apply conservation of energy and conservation of linear momentum to the motion of the two spheres.

SET UP: Denote the 50.0-kg sphere by a subscript 1 and the 100-kg sphere by a subscript 2.

EXECUTE: **(a)** Linear momentum is conserved because we are ignoring all other forces, that is, the net external force on the system is zero. Hence, $m_1v_1 = m_2v_2$.

(b) (i) From the work-energy theorem in the form $K_i + U_i = K_f + U_f$, with the initial kinetic energy $K_i = 0$ and $U = -G\dfrac{m_1m_2}{r}$, $Gm_1m_2\left[\dfrac{1}{r_f} - \dfrac{1}{r_i}\right] = \dfrac{1}{2}(m_1v_1^2 + m_2v_2^2)$. Using the conservation of momentum relation $m_1v_1 = m_2v_2$ to eliminate v_2 in favor of v_1 and simplifying yields $v_1^2 = \dfrac{2Gm_2^2}{m_1+m_2}\left[\dfrac{1}{r_f} - \dfrac{1}{r_i}\right]$, with a similar expression for v_2. Substitution of numerical values gives $v_1 = 1.49\times10^{-5}$ m/s, $v_2 = 7.46\times10^{-6}$ m/s.

(ii) The magnitude of the relative velocity is the sum of the speeds, 2.24×10^{-5} m/s.

(c) The distance the centers of the spheres travel $(x_1$ and $x_2)$ is proportional to their acceleration, and $\dfrac{x_1}{x_2} = \dfrac{a_1}{a_2} = \dfrac{m_2}{m_1}$, or $x_1 = 2x_2$. When the spheres finally make contact, their centers will be a distance of $2r$ apart, or $x_1 + x_2 + 2r = 40$ m, or $2x_2 + x_2 + 2r = 40$ m. Thus, $x_2 = 40/3\text{ m} - 2r/3$, and $x_1 = 80/3\text{ m} - 4r/3$. The point of contact of the surfaces is $80/3\text{ m} - r/3 = 26.6$ m from the initial position of the center of the 50.0-kg sphere.

EVALUATE: The result $x_1/x_2 = 2$ can also be obtained from the conservation of momentum result that $\dfrac{v_1}{v_2} = \dfrac{m_2}{m_1}$, at every point in the motion.

13.50. **IDENTIFY:** The information about Europa allows us to evaluate g at the surface of Europa. Since there is no atmosphere, $p_0 = 0$ at the surface. The pressure at depth h is $p = \rho gh$. The inward force on the window is $F_\perp = pA$.

SET UP: $g = \dfrac{Gm}{R^2}$, where $G = 6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$. $R = 1.569\times10^6$ m. Assume the ocean water has density $\rho = 1.00\times10^3\ \text{kg/m}^3$.

EXECUTE: $g=\dfrac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(4.8\times10^{22}\ \text{kg})}{(1.569\times10^6\ \text{m})^2}=1.30\ \text{m/s}^2$. The maximum pressure at the window is $p=\dfrac{9750\ \text{N}}{(0.250\ \text{m})^2}=1.56\times10^5\ \text{Pa}$. $p=\rho gh$ so $h=\dfrac{1.56\times10^5\ \text{Pa}}{(1.00\times10^3\ \text{kg/m}^3)(1.30\ \text{m/s}^2)}=120\ \text{m}$.

EVALUATE: 9750 N is the inward force exerted by the surrounding water. This will also be the net force on the window if the pressure inside the submarine is essentially zero.

13.51. **IDENTIFY** and **SET UP:** **(a)** To stay above the same point on the surface of the earth the orbital period of the satellite must equal the orbital period of the earth:

$T=1\ \text{d}(24\ \text{h/1 d})(3600\ \text{s/1 h})=8.64\times10^4\ \text{s}$

Eq. (13.14) gives the relation between the orbit radius and the period:

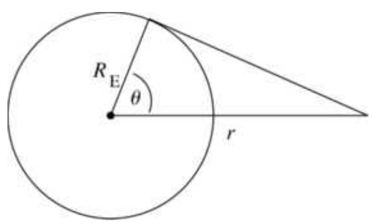
EXECUTE: $T=\dfrac{2\pi r^{3/2}}{\sqrt{Gm_E}}$ and $T^2=\dfrac{4\pi^2r^3}{Gm_E}$

$$r=\left(\frac{T^2Gm_E}{4\pi^2}\right)^{1/3}=\left(\frac{(8.64\times10^4\ \text{s})^2(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}{4\pi^2}\right)^{1/3}=4.23\times10^7\ \text{m}$$

This is the radius of the orbit; it is related to the height h above the earth's surface and the radius R_E of the earth by $r=h+R_E$. Thus $h=r-R_E=4.23\times10^7\ \text{m}-6.38\times10^6\ \text{m}=3.59\times10^7\ \text{m}$.

EVALUATE: The orbital speed of the geosynchronous satellite is $2\pi r/T=3080\ \text{m/s}$. The altitude is much larger and the speed is much less than for the satellite in Example 13.6.

(b) Consider Figure 13.51.



$$\cos\theta=\frac{R_E}{r}=\frac{6.38\times10^6\ \text{m}}{4.23\times10^7\ \text{m}}$$
$$\theta=81.3^\circ$$

Figure 13.51

A line from the satellite is tangent to a point on the earth that is at an angle of 81.3° above the equator. The sketch shows that points at higher latitudes are blocked by the earth from viewing the satellite.

13.52. **IDENTIFY:** Apply Eq. (13.12) to relate the orbital period T and M_P, the planet's mass, and then use Eq. (13.2) applied to the planet to calculate the astronaut's weight.

SET UP: The radius of the orbit of the lander is $5.75\times10^5\ \text{m}+4.80\times10^6\ \text{m}$.

EXECUTE: From Eq. (13.14), $T^2=\dfrac{4\pi^2r^3}{GM_P}$ and

$$M_P=\frac{4\pi^2r^3}{GT^2}=\frac{4\pi^2(5.75\times10^5\ \text{m}+4.80\times10^6\ \text{m})^3}{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.8\times10^3\ \text{s})^2}=2.731\times10^{24}\ \text{kg},$$

or about half the earth's mass. Now we can find the astronaut's weight on the surface from Eq. (13.2). (The landing on the north pole removes any need to account for centripetal acceleration.)

$$w=\frac{GM_Pm_a}{r_P^2}=\frac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(2.731\times10^{24}\ \text{kg})(85.6\ \text{kg})}{(4.80\times10^6\ \text{m})^2}=677\ \text{N}.$$

EVALUATE: At the surface of the earth the weight of the astronaut would be 839 N.

13.53. **IDENTIFY:** From Example 13.5, the escape speed is $v=\sqrt{\dfrac{2GM}{R}}$. Use $\rho=M/V$ to write this expression in terms of ρ.

SET UP: For a sphere $V = \frac{4}{3}\pi R^3$.

EXECUTE: In terms of the density ρ, the ratio M/R is $(4\pi/3)\rho R^2$, and so the escape speed is

$v = \sqrt{(8\pi/3)(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(2500\ \text{kg/m}^3)(150\times10^3\ \text{m})^2} = 177\ \text{m/s}.$

EVALUATE: This is much less than the escape speed for the earth, 11,200 m/s.

13.54. IDENTIFY: From Example 13.5, the escape speed is $v = \sqrt{\dfrac{2GM}{R}}$. Use $\rho = M/V$ to write this expression in terms of ρ. On earth, the height *h* you can jump is related to your jump speed by $v = \sqrt{2gh}$. For part (b), apply Eq. (13.4) to Europa.

SET UP: For a sphere $V = \frac{4}{3}\pi R^3$

EXECUTE: **(a)** $\rho = M/\left(\frac{4}{3}\pi R^3\right)$, so the escape speed can be written as $v = \sqrt{\dfrac{8\pi G\rho R^2}{3}}$. Equating the two expressions for *v* and squaring gives $2gh = \dfrac{8\pi}{3}\rho G R^2$, or $R^2 = \dfrac{3}{4\pi}\dfrac{gh}{\rho G}$, where $g = 9.80\ \text{m/s}^2$ is for the surface of the earth, not the asteroid. Estimate $h = 1$ m (variable for different people, of course), $R = 3.7$ km.

(b) For Europa, $g = \dfrac{GM}{R^2} = \dfrac{4\pi\rho RG}{3}$.

$$\rho = \frac{3g}{4\pi RG} = \frac{3(1.33\ \text{m/s}^2)}{4\pi(1.57\times10^6\ \text{m})(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)} = 3.03\times10^3\ \text{kg/m}^3.$$

EVALUATE: The earth has average density $5500\ \text{kg/m}^3$. The average density of Europa is about half that of the earth but a little larger than the average density of most asteroids.

13.55. IDENTIFY and **SET UP:** The observed period allows you to calculate the angular velocity of the satellite relative to you. You know your angular velocity as you rotate with the earth, so you can find the angular velocity of the satellite in a space-fixed reference frame. $v = r\omega$ gives the orbital speed of the satellite and Newton's second law relates this to the orbit radius of the satellite.

EXECUTE: **(a)** The satellite is revolving west to east, in the same direction the earth is rotating. If the angular speed of the satellite is ω_s and the angular speed of the earth is ω_E, the angular speed ω_{rel} of the satellite relative to you is $\omega_{rel} = \omega_s - \omega_E$.

$\omega_{rel} = (1\ \text{rev})/(12\ \text{h}) = \left(\frac{1}{12}\right)$ rev/h

$\omega_E = \left(\frac{1}{24}\right)$ rev/h

$\omega_s = \omega_{rel} + \omega_E = \left(\frac{1}{8}\right)$ rev/h $= 2.18\times10^{-4}$ rad/s

$\Sigma\vec{F} = m\vec{a}$ says $G\dfrac{mm_E}{r^2} = m\dfrac{v^2}{r}$

$v^2 = \dfrac{Gm_E}{r}$ and with $v = r\omega$ this gives $r^3 = \dfrac{Gm_E}{\omega^2}$; $r = 2.03\times10^7$ m

This is the radius of the satellite's orbit. Its height *h* above the surface of the earth is $h = r - R_E = 1.39\times10^7$ m.

(b) Now the satellite is revolving opposite to the rotation of the earth. If west to east is positive, then

$\omega_{rel} = \left(-\frac{1}{12}\right)$ rev/h

$\omega_s = \omega_{rel} + \omega_E = \left(-\frac{1}{24}\right)$ rev/h $= -7.27\times10^{-5}$ rad/s

$r^3 = \dfrac{Gm_E}{\omega^2}$ gives $r = 4.22\times10^7$ m and $h = 3.59\times10^7$ m

EVALUATE: In part (a) the satellite is revolving faster than the earth's rotation and in part (b) it is revolving slower. Slower *v* and ω means larger orbit radius *r*.

13.56. **IDENTIFY:** Apply the law of gravitation to the astronaut at the north pole to calculate the mass of planet. Then apply $\Sigma\vec{F} = m\vec{a}$ to the astronaut, with $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$, toward the center of the planet, to calculate the period T. Apply Eq. (13.12) to the satellite in order to calculate its orbital period.

SET UP: Get radius of X: $\frac{1}{4}(2\pi R) = 18{,}850$ km and $R = 1.20\times10^7$ m. Astronaut mass:

$$m = \frac{w}{g} = \frac{943\text{ N}}{9.80\text{ m/s}^2} = 96.2\text{ kg}.$$

EXECUTE: $\dfrac{GmM_X}{R^2} = w$, where $w = 915.0$ N.

$$M_X = \frac{mg_X R^2}{Gm} = \frac{(915\text{ N})(1.20\times10^7\text{ m})^2}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(96.2\text{ kg})} = 2.05\times10^{25}\text{ kg}$$

Apply Newton's second law to astronaut on a scale at the equator of X. $F_{\text{grav}} - F_{\text{scale}} = ma_{\text{rad}}$, so

$$F_{\text{grav}} - F_{\text{scale}} = \frac{4\pi^2 mR}{T^2}.\quad 915.0\text{ N} - 850.0\text{ N} = \frac{4\pi^2(96.2\text{ kg})(1.20\times10^7\text{ m})}{T^2}$$ and

$$T = 2.65\times10^4\text{ s}\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 7.36\text{ h}.$$

(b) For the satellite, $T = \sqrt{\dfrac{4\pi^2 r^3}{Gm_X}} = \sqrt{\dfrac{4\pi^2(1.20\times10^7\text{ m} + 2.0\times10^6\text{ m})^3}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(2.05\times10^{25}\text{ kg})}} = 8.90\times10^3\text{ s} = 2.47$ hours.

EVALUATE: The acceleration of gravity at the surface of the planet is $g_X = \dfrac{915.0\text{ N}}{96.2\text{ kg}} = 9.51\text{ m/s}^2$, similar to the value on earth. The radius of the planet is about twice that of earth. The planet rotates more rapidly than earth and the length of a day is about one-third what it is on earth.

13.57. **IDENTIFY:** Use $g = \dfrac{Gm_E}{R_E^2}$ and follow the procedure specified in the problem.

SET UP: $R_E = 6.38\times10^6$ m

EXECUTE: The fractional error is $1 - \dfrac{mgh}{Gmm_E(1/R_E - 1/(R_E + h))} = 1 - \dfrac{g}{Gm_E}(R_E + h)(R_E)$.

Using Eq. (13.4) for g the fractional difference is $1 - (R_E + h)/R_E = -h/R_E$, so if the fractional difference is -1%, $h = (0.01)R_E = 6.4\times10^4$ m.

EVALUATE: For $h = 1$ km, the fractional error is only 0.016%. Eq. (7.2) is very accurate for the motion of objects near the earth's surface.

13.58. **IDENTIFY:** Use the measurements of the motion of the rock to calculate g_M, the value of g on Mongo. Then use this to calculate the mass of Mongo. For the ship, $F_g = ma_{\text{rad}}$ and $T = \dfrac{2\pi r}{v}$.

SET UP: Take $+y$ upward. When the stone returns to the ground its velocity is 12.0 m/s, downward.

$g_M = G\dfrac{m_M}{R_M^2}$. The radius of Mongo is $R_M = \dfrac{c}{2\pi} = \dfrac{2.00\times10^8\text{ m}}{2\pi} = 3.18\times10^7$ m. The ship moves in an orbit of radius $r = 3.18\times10^7\text{ m} + 3.00\times10^7\text{ m} = 6.18\times10^7$ m.

EXECUTE: **(a)** $v_{0y} = +12.0$ m/s, $v_y = -12.0$ m/s, $a_y = -g_M$ and $t = 6.00$ s. $v_y = v_{0y} + a_y t$ gives

$$-g_M = \frac{v_y - v_{0y}}{t} = \frac{-12.0\text{ m/s} - 12.0\text{ m/s}}{6.00\text{ s}}$$ and $g_M = 4.00\text{ m/s}^2$.

$$m_M = \frac{g_M R_M^2}{G} = \frac{(4.00\text{ m/s}^2)(3.18\times10^7\text{ m})^2}{6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2} = 6.06\times10^{25}\text{ kg}$$

(b) $F_g = ma_{rad}$ gives $G\dfrac{m_M m}{r^2} = m\dfrac{v^2}{r}$ and $v^2 = \dfrac{Gm_M}{r}$.

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{Gm_M}} = \frac{2\pi r^{3/2}}{\sqrt{Gm_M}} = \frac{2\pi(6.18\times10^7\ \text{m})^{3/2}}{\sqrt{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(6.06\times10^{25}\ \text{kg})}}$$

$T = 4.80\times10^4\ \text{s} = 13.3\ \text{h}$

EVALUATE: $R_M = 5.0R_E$ and $m_M = 10.2m_E$, so $g_M = \dfrac{10.2}{(5.0)^2}g_E = 0.408g_E$, which agrees with the value calculated in part (a).

13.59. **IDENTIFY:** The free-fall time of the rock will give us the acceleration due to gravity at the surface of the planet. Applying Newton's second law and the law of universal gravitation will give us the mass of the planet since we know its radius.

SET UP: For constant acceleration, $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$. At the surface of the planet, Newton's second law gives $m_{rock}g = \dfrac{Gm_{rock}m_p}{R_p^2}$.

EXECUTE: First find $a_y = g$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$. $a_y = \dfrac{2(y-y_0)}{t^2} = \dfrac{2(1.90\ \text{m})}{(0.480\ \text{s})^2} = 16.49\ \text{m/s}^2 = g$.

$g = 16.49\ \text{m/s}^2$. $m_p = \dfrac{gR_p^2}{G} = \dfrac{(16.49\ \text{m/s})(8.60\times10^7\ \text{m})^2}{6.674\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2} = 1.83\times10^{27}\ \text{kg}$.

EVALUATE: The planet's mass is over 100 times that of the earth, which is reasonable since it is larger (in size) than the earth yet has a greater acceleration due to gravity at its surface.

13.60. **IDENTIFY:** Apply Eq. (13.9) to the particle-earth and particle-moon systems.

SET UP: When the particle is a distance *r* from the center of the earth, it is a distance $R_{EM} - r$ from the center of the moon.

EXECUTE: **(a)** The total gravitational potential energy in this model is $U = -Gm\left[\dfrac{m_E}{r} + \dfrac{m_M}{R_{EM} - r}\right]$.

(b) The point where the net gravitational force vanishes is $r = \dfrac{R_{EM}}{1+\sqrt{m_M/m_E}} = 3.46\times10^8\ \text{m}$. Using this value for *r* in the expression in part (a) and the work-energy theorem, including the initial potential energy of $-Gm(m_E/R_E + m_M/(R_{EM} - R_E))$ gives $11.1\ \text{km/s}$.

(c) The final distance from the earth is the Earth-moon distance minus the radius of the moon, or $3.823\times10^8\ \text{m}$. From the work-energy theorem, the rocket impacts the moon with a speed of $2.9\ \text{km/s}$.

EVALUATE: The spacecraft has a greater gravitational potential energy at the surface of the moon than at the surface of the earth, so it reaches the surface of the moon with a speed that is less than its launch speed on earth.

13.61. **IDENTIFY** and **SET UP:** Use Eq. (13.2) to calculate the gravity force at each location. For the top of Mount Everest write $r = h + R_E$ and use the fact that $h \ll R_E$ to obtain an expression for the difference in the two forces.

EXECUTE: At Sacramento, the gravity force on you is $F_1 = G\dfrac{mm_E}{R_E^2}$.

At the top of Mount Everest, a height of $h = 8800\ \text{m}$ above sea level, the gravity force on you is

$$F_2 = G\frac{mm_E}{(R_E + h)^2} = G\frac{mm_E}{R_E^2(1 + h/R_E)^2}$$

$$(1 + h/R_E)^{-2} \approx 1 - \frac{2h}{R_E},\quad F_2 = F_1\left(1 - \frac{2h}{R_E}\right)$$

$$\frac{F_1 - F_2}{F_1} = \frac{2h}{R_E} = 0.28\%$$

EVALUATE: The change in the gravitational force is very small, so for objects near the surface of the earth it is a good approximation to treat it as a constant.

13.62. **IDENTIFY:** The 0.100 kg sphere has gravitational potential energy due to the other two spheres. Its mechanical energy is conserved.

SET UP: From energy conservation, $K_1 + U_1 = K_2 + U_2$, where $K = \frac{1}{2}mv^2$, and $U = -Gm_1m_2/r$.

EXECUTE: Using $K_1 + U_1 = K_2 + U_2$, we have $K_1 = 0$, $m_A = 5.00\text{ kg}$, $m_B = 10.0\text{ kg}$ and $m = 0.100\text{ kg}$.

$$U_1 = -\frac{Gmm_A}{r_{A1}} - \frac{Gmm_B}{r_{B1}} = -(6.674\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(0.100\text{ kg})\left(\frac{5.00\text{ kg}}{0.400\text{ m}} + \frac{10.0\text{ kg}}{0.600\text{ m}}\right)$$

$U_1 = -1.9466\times10^{-10}\text{ J}$.

$$U_2 = -\frac{Gmm_A}{r_{A2}} - \frac{Gmm_B}{r_{B2}} = -(6.674\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(0.100\text{ kg})\left(\frac{5.00\text{ kg}}{0.800\text{ m}} + \frac{10.0\text{ kg}}{0.200\text{ m}}\right)$$

$U_2 = -3.7541\times10^{-10}\text{ J}$. $K_2 = U_1 - U_2 = -1.9466\times10^{-10}\text{ J} - (-3.7541\times10^{-10}\text{ J}) = 1.8075\times10^{-10}\text{ J}$.

$$\frac{1}{2}mv^2 = K_2 \text{ and } v = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(1.8075\times10^{-10}\text{ J})}{0.100\text{ kg}}} = 6.01\times10^{-5}\text{ m/s}.$$

EVALUATE: The kinetic energy gained by the sphere is equal to the loss in its potential energy.

13.63. **IDENTIFY** and **SET UP:** First use the radius of the orbit to find the initial orbital speed, from Eq. (13.10) applied to the moon.

EXECUTE: $v = \sqrt{Gm/r}$ and $r = R_M + h = 1.74\times10^6\text{ m} + 50.0\times10^3\text{ m} = 1.79\times10^6\text{ m}$

$$\text{Thus } v = \sqrt{\frac{(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(7.35\times10^{22}\text{ kg})}{1.79\times10^6\text{ m}}} = 1.655\times10^3\text{ m/s}$$

After the speed decreases by 20.0 m/s it becomes $1.655\times10^3\text{ m/s} - 20.0\text{ m/s} = 1.635\times10^3\text{ m/s}$.

IDENTIFY and **SET UP:** Use conservation of energy to find the speed when the spacecraft reaches the lunar surface.

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

Gravity is the only force that does work so $W_{\text{other}} = 0$ and $K_2 = K_1 + U_1 - U_2$

EXECUTE: $U_1 = -Gm_m m/r$; $U_2 = -Gm_m m/R_m$

$\frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2 + Gmm_m(1/R_m - 1/r)$

And the mass m divides out to give $v_2 = \sqrt{v_1^2 + 2Gm_m(1/R_m - 1/r)}$

$v_2 = 1.682\times10^3\text{ m/s}(1\text{ km}/1000\text{ m})(3600\text{ s}/1\text{ h}) = 6060\text{ km/h}$

EVALUATE: After the thruster fires the spacecraft is moving too slowly to be in a stable orbit; the gravitational force is larger than what is needed to maintain a circular orbit. The spacecraft gains energy as it is accelerated toward the surface.

13.64. **IDENTIFY:** In part (a) use the expression for the escape speed that is derived in Example 13.5. In part (b) apply conservation of energy.

SET UP: $R = 4.5\times10^3\text{ m}$. In part (b) let point 1 be at the surface of the comet.

EXECUTE: **(a)** The escape speed is $v = \sqrt{\frac{2GM}{R}}$ so

$$M = \frac{Rv^2}{2G} = \frac{(4.5\times10^3\text{ m})(1.0\text{ m/s})^2}{2(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)} = 3.37\times10^{13}\text{ kg}.$$

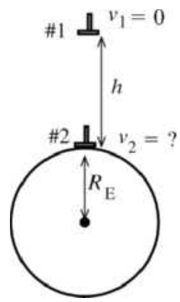
(b) (i) $K_1 = \tfrac{1}{2}mv_1^2$. $K_2 = 0.100K_1$. $U_1 = -\dfrac{GMm}{R}$; $U_2 = -\dfrac{GMm}{r}$. $K_1 + U_1 = K_2 + U_2$ gives

$\tfrac{1}{2}mv_1^2 - \dfrac{GMm}{R} = (0.100)(\tfrac{1}{2}mv_1^2) - \dfrac{GMm}{r}$. Solving for r gives

$\dfrac{1}{r} = \dfrac{1}{R} - \dfrac{0.450v_1^2}{GM} = \dfrac{1}{4.5\times10^3\ \text{m}} - \dfrac{0.450(1.0\ \text{m/s})^2}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(3.37\times10^{13}\ \text{kg})}$ and $r = 45$ km. (ii) The debris never loses all of its initial kinetic energy, but $K_2 \to 0$ as $r \to \infty$. The farther the debris are from the comet's center, the smaller is their kinetic energy.

EVALUATE: The debris will have lost 90.0% of their initial kinetic energy when they are at a distance from the comet's center of about ten times the radius of the comet.

13.65. **IDENTIFY** and **SET UP:** Apply conservation of energy. Must use Eq. (13.9) for the gravitational potential energy since h is not small compared to R_E.



As indicated in Figure 13.65, take point 1 to be where the hammer is released and point 2 to be just above the surface of the earth, so $r_1 = R_E + h$ and $r_2 = R_E$.

Figure 13.65

EXECUTE: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

Only gravity does work, so $W_{\text{other}} = 0$.

$K_1 = 0,\ \ K_2 = \tfrac{1}{2}mv_2^2$

$U_1 = -G\dfrac{mm_E}{r_1} = -\dfrac{Gmm_E}{h+R_E},\ \ U_2 = -G\dfrac{mm_E}{r_2} = -\dfrac{Gmm_E}{R_E}$

Thus, $-G\dfrac{mm_E}{h+R_E} = \dfrac{1}{2}mv_2^2 - G\dfrac{mm_E}{R_E}$

$$v_2^2 = 2Gm_E\left(\frac{1}{R_E} - \frac{1}{R_E+h}\right) = \frac{2Gm_E}{R_E(R_E+h)}(R_E + h - R_E) = \frac{2Gm_Eh}{R_E(R_E+h)}$$

$$v_2 = \sqrt{\frac{2Gm_Eh}{R_E(R_E+h)}}$$

EVALUATE: If $h \to \infty$, $v_2 \to \sqrt{2Gm_E/R_E}$, which equals the escape speed. In this limit this event is the reverse of an object being projected upward from the surface with the escape speed. If $h \ll R_E$, then $v_2 = \sqrt{2Gm_Eh/R_E^2} = \sqrt{2gh}$, the same result if Eq. (7.2) used for U.

13.66. **IDENTIFY:** In orbit the total mechanical energy of the satellite is $E = -\dfrac{Gm_Em}{2R_E}$. $U = -G\dfrac{m_Em}{r}$.

$W = E_2 - E_1$.

SET UP: $U \to 0$ as $r \to \infty$.

EXECUTE: **(a)** The energy the satellite has as it sits on the surface of the Earth is $E_1 = \dfrac{-GmM_E}{R_E}$. The energy it has when it is in orbit at a radius $R \approx R_E$ is $E_2 = \dfrac{-GmM_E}{2R_E}$. The work needed to put it in orbit is the difference between these: $W = E_2 - E_1 = \dfrac{GmM_E}{2R_E}$.

(b) The total energy of the satellite far away from the earth is zero, so the additional work needed is $0 - \left(\dfrac{-GmM_E}{2R_E}\right) = \dfrac{GmM_E}{2R_E}$.

EVALUATE: **(c)** The work needed to put the satellite into orbit was the same as the work needed to put the satellite from orbit to the edge of the universe.

13.67. **IDENTIFY:** At the escape speed, $E = K + U = 0$.

SET UP: At the surface of the earth the satellite is a distance $R_E = 6.38\times10^6$ m from the center of the earth and a distance $R_{ES} = 1.50\times10^{11}$ m from the sun. The orbital speed of the earth is $\dfrac{2\pi R_{ES}}{T}$, where $T = 3.156\times10^7$ s is the orbital period. The speed of a point on the surface of the earth at an angle ϕ from the equator is $v = \dfrac{2\pi R_E \cos\phi}{T}$, where $T = 86{,}400$ s is the rotational period of the earth.

EXECUTE: **(a)** The escape speed will be $v = \sqrt{2G\left[\dfrac{m_E}{R_E} + \dfrac{m_s}{R_{ES}}\right]} = 4.35\times10^4$ m/s. Making the simplifying assumption that the direction of launch is the direction of the earth's motion in its orbit, the speed relative to the center of the earth is $v - \dfrac{2\pi R_{ES}}{T} = 4.35\times10^4\ \text{m/s} - \dfrac{2\pi(1.50\times10^{11}\ \text{m})}{(3.156\times10^7\ \text{s})} = 1.37\times10^4$ m/s.

(b) The rotational speed at Cape Canaveral is $\dfrac{2\pi(6.38\times10^6\ \text{m})\cos 28.5^\circ}{86{,}400\ \text{s}} = 4.09\times10^2$ m/s, so the speed relative to the surface of the earth is 1.33×10^4 m/s.

(c) In French Guiana, the rotational speed is 4.63×10^2 m/s, so the speed relative to the surface of the earth is 1.32×10^4 m/s.

EVALUATE: The orbital speed of the earth is a large fraction of the escape speed, but the rotational speed of a point on the surface of the earth is much less.

13.68. **IDENTIFY:** From the discussion of Section 13.6, the force on a point mass at a distance *r* from the center of a spherically symmetric mass distribution is the same as though we removed all the mass at points farther than *r* from the center and concentrated all the remaining mass at the center.

SET UP: The mass *M* of a hollow sphere of density ρ, inner radius R_1 and outer radius R_2 is $M = \rho\frac{4}{3}\pi(R_2^3 - R_1^3)$. From Figure 13.9 in the textbook, the inner core has outer radius 1.2×10^6 m, inner radius zero and density $1.3\times10^4\ \text{kg/m}^3$. The outer core has inner radius 1.2×10^6 m, outer radius 3.6×10^6 m and density $1.1\times10^4\ \text{kg/m}^3$. The total mass of the earth is $m_E = 5.97\times10^{24}$ kg and its radius is $R_E = 6.38\times10^6$ m.

EXECUTE: **(a)** $F_g = G\dfrac{m_E m}{R_E^2} = mg = (10.0\ \text{kg})(9.80\ \text{m/s}^2) = 98.0\ \text{N}$.

(b) The mass of the inner core is $m_{\text{inner}} = \rho_{\text{inner}}\frac{4}{3}\pi(R_2^3 - R_1^3) = (1.3\times10^4\ \text{kg/m}^3)\frac{4}{3}\pi(1.2\times10^6\ \text{m})^3 = 9.4\times10^{22}$ kg. The mass of the outer core is $m_{\text{outer}} = (1.1\times10^4\ \text{kg/m}^3)\frac{4}{3}\pi([3.6\times10^6\ \text{m}]^3 - [1.2\times10^6\ \text{m}]^3) = 2.1\times10^{24}$ kg. Only the inner and outer cores contribute to the force.

$$F_g = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(9.4\times10^{22}\ \text{kg} + 2.1\times10^{24}\ \text{kg})(10.0\ \text{kg})}{(3.6\times10^6\ \text{m})^2} = 110\ \text{N}.$$

(c) Only the inner core contributes to the force and

$$F_g = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(9.4\times10^{22}\ \text{kg})(10.0\ \text{kg})}{(1.2\times10^6\ \text{m})^2} = 44\ \text{N}.$$

(d) At $r = 0,\ F_g = 0.$

EVALUATE: In this model the earth is spherically symmetric but not uniform, so the result of Example 13.10 doesn't apply. In particular, the force at the surface of the outer core is greater than the force at the surface of the earth.

13.69. **IDENTIFY:** Eq. (13.12) relates orbital period and orbital radius for a circular orbit.

SET UP: The mass of the sun is $M = 1.99\times10^{30}$ kg.

EXECUTE: **(a)** The period of the asteroid is $T = \dfrac{2\pi a^{3/2}}{\sqrt{GM}}$. Inserting (i) 3×10^{11} m for a gives 2.84 y and (ii) 5×10^{11} m gives a period of 6.11 y.

(b) If the period is 5.93 y, then $a = 4.90\times10^{11}$m.

(c) This happens because $0.4 = 2/5,$ another ratio of integers. So once every 5 orbits of the asteroid and 2 orbits of Jupiter, the asteroid is at its perijove distance. Solving when $T = 4.74\ y,\ a = 4.22\times10^{11}$ m.

EVALUATE: The orbit radius for Jupiter is 7.78×10^{11} m and for Mars it is 2.28×10^{11} m. The asteroid belt lies between Mars and Jupiter. The mass of Jupiter is about 3000 times that of Mars, so the effect of Jupiter on the asteroids is much larger.

13.70. **IDENTIFY:** Apply the work-energy relation in the form $W = \Delta E,$ where $E = K + U.$ The speed v is related to the orbit radius by Eq. (13.10).

SET UP: $m_E = 5.97\times10^{24}$ kg

EXECUTE: **(a)** In moving to a lower orbit by whatever means, gravity does positive work, and so the speed does increase.

(b) $v = (Gm_E)^{1/2}r^{-1/2},$ so $\Delta v = (Gm_E)^{1/2}\left(-\dfrac{-\Delta r}{2}\right)r^{-3/2} = \left(\dfrac{\Delta r}{2}\right)\sqrt{\dfrac{Gm_E}{r^3}}.$ Note that a positive Δr is given as a decrease in radius. Similarly, the kinetic energy is $K = (1/2)mv^2 = (1/2)Gm_E m/r,$ and so

$\Delta K = (1/2)(Gm_E m/r^2)\Delta r$ and $\Delta U = -(Gm_E m/r^2)\Delta r.$

$W = \Delta U + \Delta K = -(Gm_E m/2r^2)\,\Delta r$

(c) $v = \sqrt{Gm_E/r} = 7.72\times10^3$ m/s, $\Delta v = (\Delta r/2)\sqrt{Gm_E/r^3} = 28.9$ m/s, $E = -Gm_E m/2r = -8.95\times10^{10}$ J (from Eq. (13.15)), $\Delta K = (Gm_E m/2r^2)(\Delta r) = 6.70\times10^8$ J, $\Delta U = -2\Delta K = -1.34\times10^9$ J, and

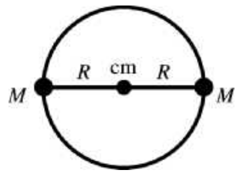
$W = -\Delta K = -6.70\times10^8$ J.

(d) As the term "burns up" suggests, the energy is converted to heat or is dissipated in the collisions of the debris with the ground.

EVALUATE: When r decreases, K increases and U decreases (becomes more negative).

13.71. **IDENTIFY:** Use Eq. (13.2) to calculate F_g. Apply Newton's second law to circular motion of each star to find the orbital speed and period. Apply the conservation of energy expression, Eq. (7.13), to calculate the energy input (work) required to separate the two stars to infinity.

(a) SET UP: The cm is midway between the two stars since they have equal masses. Let R be the orbit radius for each star, as sketched in Figure 13.71.



The two stars are separated by a distance $2R$, so
$F_g = GM^2/(2R)^2 = GM^2/4R^2$

Figure 13.71

(b) EXECUTE: $F_g = ma_{rad}$

$GM^2/4R^2 = M(v^2/R)$ so $v = \sqrt{GM/4R}$

And $T = 2\pi R/v = 2\pi R\sqrt{4R/GM} = 4\pi\sqrt{R^3/GM}$

(c) SET UP: Apply $K_1 + U_1 + W_{other} = K_2 + U_2$ to the system of the two stars. Separate to infinity implies $K_2 = 0$ and $U_2 = 0$.

EXECUTE: $K_1 = \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = 2\left(\frac{1}{2}M\right)(GM/4R) = GM^2/4R$

$U_1 = -GM^2/2R$

Thus the energy required is $W_{other} = -(K_1 + U_1) = -(GM^2/4R - GM^2/2R) = GM^2/4R.$

EVALUATE: The closer the stars are and the greater their mass, the larger their orbital speed, the shorter their orbital period and the greater the energy required to separate them.

13.72. IDENTIFY: In the center of mass coordinate system, $r_{cm} = 0$. Apply $\vec{F} = m\vec{a}$ to each star, where F is the gravitational force of one star on the other and $a = a_{rad} = \dfrac{4\pi^2 R}{T^2}$.

SET UP: $v = \dfrac{2\pi R}{T}$ allows R to be calculated from v and T.

EXECUTE: **(a)** The radii R_1 and R_2 are measured with respect to the center of mass, and so $M_1R_1 = M_2R_2$, and $R_1/R_2 = M_2/M_1$.

(b) The forces on each star are equal in magnitude, so the product of the mass and the radial accelerations are equal: $\dfrac{4\pi^2 M_1R_1}{T_1^2} = \dfrac{4\pi^2 M_2R_2}{T_2^2}$. From the result of part (a), the numerators of these expressions are equal, and so the denominators are equal, and the periods are the same. To find the period in the symmetric form desired, there are many possible routes. An elegant method, using a bit of hindsight, is to use the above expressions to relate the periods to the force $F_g = \dfrac{GM_1M_2}{(R_1 + R_2)^2}$, so that equivalent expressions for the period are $M_2T^2 = \dfrac{4\pi^2 R_1(R_1 + R_2)^2}{G}$ and $M_1T^2 = \dfrac{4\pi^2 R_2(R_1 + R_2)^2}{G}$. Adding the expressions gives

$(M_1 + M_2)T^2 = \dfrac{4\pi^2(R_1 + R_2)^3}{G}$ or $T = \dfrac{2\pi(R_1 + R_2)^{3/2}}{\sqrt{G(M_1 + M_2)}}$.

(c) First we must find the radii of each orbit given the speed and period data. In a circular orbit, $v = \dfrac{2\pi R}{T}$, or $R = \dfrac{vT}{2\pi}$. Thus $R_\alpha = \dfrac{(36\times 10^3 \text{ m/s})(137 \text{ d})(86{,}400 \text{ s/d})}{2\pi} = 6.78\times 10^{10}$ m and

$R_\beta = \dfrac{(12\times 10^3 \text{ m/s})(137 \text{ d})(86{,}400 \text{ s/d})}{2\pi} = 2.26\times 10^{10}$ m. Now find the sum of the masses.

$(M_\alpha + M_\beta) = \dfrac{4\pi^2(R_\alpha + R_\beta)^3}{T^2 G}$. Inserting the values of T and the radii gives

$$(M_\alpha + M_\beta) = \frac{4\pi^2(6.78\times10^{10}\,\text{m} + 2.26\times10^{10}\ \text{m})^3}{[(137\ \text{d})(86{,}400\ \text{s/d})]^2(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)} = 3.12\times10^{30}\ \text{kg. Since}$$

$M_\beta = M_\alpha R_\alpha / R_\beta = 3M_\alpha$, $4M_\alpha = 3.12\times10^{30}$ kg, or $M_\alpha = 7.80\times10^{29}$ kg, and $M_\beta = 2.34\times10^{30}$ kg.

(d) Let α refer to the star and β refer to the black hole. Use the relationships derived in parts (a) and (b):

$R_\beta = (M_\alpha / M_\beta)R_\alpha = (0.67/3.8)R_\alpha = (0.176)R_\alpha$, $R_\alpha + R_\beta = \sqrt[3]{\dfrac{(M_\alpha + M_\beta)T^2 G}{4\pi^2}}$. For Monocerotis,

inserting the values for M and T gives $R_\alpha = 1.9\times10^9$ m, $v_\alpha = 4.4\times10^2$ km/s and for the black hole $R_\beta = 34\times10^8$ m, $v_\beta = 77$ km/s.

EVALUATE: Since T is the same, v is smaller when R is smaller.

13.73. **IDENTIFY** and **SET UP:** Use conservation of energy, $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. The gravity force exerted by the sun is the only force that does work on the comet, so $W_{\text{other}} = 0$.

EXECUTE: $K_1 = \frac{1}{2}mv_1^2$, $v_1 = 2.0\times10^4$ m/s

$U_1 = -Gm_S m/r_1$, $r_1 = 2.5\times10^{11}$ m

$K_2 = \frac{1}{2}mv_2^2$

$U_2 = -Gm_S m/r_2$, $r_2 = 5.0\times10^{10}$ m

$\frac{1}{2}mv_1^2 - Gm_S m/r_1 = \frac{1}{2}mv_2^2 - Gm_S m/r_2$

$$v_2^2 = v_1^2 + 2Gm_S\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = v_1^2 + 2Gm_S\left(\frac{r_1 - r_2}{r_1 r_2}\right)$$

$v_2 = 6.8\times10^4$ m/s

EVALUATE: The comet has greater speed when it is closer to the sun.

13.74. **IDENTIFY** and **SET UP:** Apply Eq. (12.6) and solve for g.
Then use Eq. (13.4) to relate g to the mass of the planet.

EXECUTE: $p - p_0 = \rho g d$.

This expression gives that $g = (p - p_0)/\rho d = (p - p_0)V/md$.

But also $g = Gm_p/R^2$. (Eq. (13.4) applied to the planet rather than to earth.)

Equating these two expressions for g gives $Gm_p/R^2 = (p - p_0)V/md$ and $m_p = (p - p_0)VR^2/Gmd$.

EVALUATE: The greater p is at a given depth, the greater g is for the planet and greater g means greater m_p.

13.75. **IDENTIFY:** Follow the procedure outlined in part (b). For a spherically symmetric object, with total mass m and radius r, at points on the surface of the object, $g(r) = Gm/r^2$.

SET UP: The earth has mass $m_E = 5.97\times10^{24}$ kg. If $g(r)$ is a maximum at $r = r_{\max}$, then $\dfrac{dg}{dr} = 0$ for $r = r_{\max}$.

EXECUTE: **(a)** At $r = 0$, the model predicts $\rho = A = 12{,}700$ kg/m^3 and at $r = R$, the model predicts $\rho = A - BR = 12{,}700\ \text{kg/m}^3 - (1.50\times10^{-3}\ \text{kg/m}^4)(6.37\times10^6\ \text{m}) = 3.15\times10^3\ \text{kg/m}^3$.

(b) and **(c)** $M = \int dm = 4\pi\int_0^R [A - Br]r^2 dr = 4\pi\left[\dfrac{AR^3}{3} - \dfrac{BR^4}{4}\right] = \left(\dfrac{4\pi R^3}{3}\right)\left[A - \dfrac{3BR}{4}\right]$.

$$M = \left(\frac{4\pi(6.37\times10^6\ \text{m})^3}{3}\right)\left[12{,}700\ \text{kg/m}^3 - \frac{3(1.50\times10^{-3}\ \text{kg/m}^4)(6.37\times10^6\ \text{m})}{4}\right] = 5.99\times10^{24}\ \text{kg}$$

which is within 0.36% of the earth's mass.

(d) If $m(r)$ is used to denote the mass contained in a sphere of radius $r,$ then $g = Gm(r)/r^2$. Using the same integration as that in part (b), with an upper limit of r instead of R gives the result.
(e) $g = 0$ at $r = 0$, and g at $r = R$ is

$$g = Gm(R)/R^2 = (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.99\times10^{24}\ \text{kg})/(6.37\times10^6\ \text{m})^2 = 9.85\ \text{m/s}^2.$$

(f) $\dfrac{dg}{dr} = \left(\dfrac{4\pi G}{3}\right)\dfrac{d}{dr}\left[Ar - \dfrac{3Br^2}{4}\right] = \left(\dfrac{4\pi G}{3}\right)\left[A - \dfrac{3Br}{2}\right]$. Setting this equal to zero gives

$r = 2A/3B = 5.64\times10^6$ m, and at this radius $g = \left(\dfrac{4\pi G}{3}\right)\left(\dfrac{2A}{3B}\right)\left[A - \left(\dfrac{3}{4}\right)B\left(\dfrac{2A}{3B}\right)\right] = \dfrac{4\pi GA^2}{9B}$.

$$g = \frac{4\pi(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(12,700\ \text{kg/m}^3)^2}{9(1.50\times10^{-3}\ \text{kg/m}^4)} = 10.02\ \text{m/s}^2.$$

EVALUATE: If the earth were a uniform sphere of density ρ, then $g(r) = \dfrac{\rho V(r)}{r^2} = \left(\dfrac{4\pi\rho G}{3}\right)r$, the same as setting $B = 0$ and $A = \rho$ in $g(r)$ in part (d). If r_{max} is the value of r in part (f) where $g(r)$ is a maximum, then $r_{\text{max}}/R = 0.885$. For a uniform sphere, $g(r)$ is maximum at the surface.

13.76. **IDENTIFY:** Follow the procedure outlined in part (a).

SET UP: The earth has mass $M = 5.97\times10^{24}$ kg and radius $R = 6.38\times10^6$ m. Let $g_\text{S} = 9.80\ \text{m/s}^2$.

EXECUTE: **(a)** Eq. (12.4), with the radius r instead of height y, becomes $dp = -\rho g(r)\,dr = -\rho g_\text{S}(r/R)dr$. This form shows that the pressure decreases with increasing radius. Integrating, with $p = 0$ at $r = R$,

$$p = -\frac{\rho g_\text{S}}{R}\int_R^r r\,dr = \frac{\rho g_\text{S}}{R}\int_r^R r\,dr = \frac{\rho g_\text{S}}{2R}(R^2 - r^2).$$

(b) Using the above expression with $r = 0$ and $\rho = \dfrac{M}{V} = \dfrac{3M}{4\pi R^3}$,

$$p(0) = \frac{3(5.97\times10^{24}\ \text{kg})(9.80\ \text{m/s}^2)}{8\pi(6.38\times10^6\ \text{m})^2} = 1.71\times10^{11}\ \text{Pa}.$$

(c) While the same order of magnitude, this is not in very good agreement with the estimated value. In more realistic density models (see Problem 13.75), the concentration of mass at lower radii leads to a higher pressure.

EVALUATE: In this model, the pressure at the center of the earth is about 10^6 times what it is at the surface.

13.77. **(a) IDENTIFY** and **SET UP:** Use Eq. (13.17), applied to the satellites orbiting the earth rather than the sun.

EXECUTE: Find the value of a for the elliptical orbit:

$2a = r_\text{a} + r_\text{p} = R_\text{E} + h_\text{a} + R_\text{E} + h_\text{p}$, where h_a and h_p are the heights at apogee and perigee, respectively.

$$a = R_\text{E} + (h_\text{a} + h_\text{p})/2$$

$$a = 6.38\times10^6\ \text{m} + (400\times10^3\ \text{m} + 4000\times10^3\ \text{m})/2 = 8.58\times10^6\ \text{m}$$

$$T = \frac{2\pi a^{3/2}}{\sqrt{GM_\text{E}}} = \frac{2\pi(8.58\times10^6\ \text{m})^{3/2}}{\sqrt{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}} = 7.91\times10^3\ \text{s}$$

(b) Conservation of angular momentum gives $r_\text{a}v_\text{a} = r_\text{p}v_\text{p}$

$$\frac{v_\text{p}}{v_\text{a}} = \frac{r_\text{a}}{r_\text{p}} = \frac{6.38\times10^6\ \text{m} + 4.00\times10^6\ \text{m}}{6.38\times10^6\ \text{m} + 4.00\times10^5\ \text{m}} = 1.53$$

(c) Conservation of energy applied to apogee and perigee gives $K_\text{a} + U_\text{a} = K_\text{p} + U_\text{p}$

$$\tfrac{1}{2}mv_\text{a}^2 - Gm_\text{E}m/r_\text{a} = \tfrac{1}{2}mv_\text{p}^2 - Gm_\text{E}m/r_\text{p}$$

$$v_\text{p}^2 - v_\text{a}^2 = 2Gm_\text{E}(1/r_\text{p} - 1/r_\text{a}) = 2Gm_\text{E}(r_\text{a} - r_\text{p})/r_\text{a}r_\text{p}$$

But $v_p = 1.532 v_a$, so $1.347 v_a^2 = 2Gm_E(r_a - r_p)/r_a r_p$

$v_a = 5.51\times10^3$ m/s, $v_p = 8.43\times10^3$ m/s

(d) Need v so that $E = 0$, where $E = K + U$.

at perigee: $\frac{1}{2}mv_p^2 - Gm_E m/r_p = 0$

$$v_p = \sqrt{2Gm_E/r_p} = \sqrt{2(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})/6.78\times10^6\text{ m}} = 1.084\times10^4\text{ m/s}$$

This means an increase of $1.084\times10^4\text{ m/s} - 8.43\times10^3\text{ m/s} = 2.41\times10^3$ m/s.

at apogee:

$$v_a = \sqrt{2Gm_E/r_a} = \sqrt{2(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})/1.038\times10^7\text{ m}} = 8.761\times10^3\text{ m/s}$$

This means an increase of $8.761\times10^3\text{ m/s} - 5.51\times10^3\text{ m/s} = 3.25\times10^3$ m/s.

EVALUATE: Perigee is more efficient. At this point r is smaller so v is larger and the satellite has more kinetic energy and more total energy.

13.78. **IDENTIFY:** $g = \dfrac{GM}{R^2}$, where M and R are the mass and radius of the planet.

SET UP: Let m_U and R_U be the mass and radius of Uranus and let g_U be the acceleration due to gravity at its poles. The orbit radius of Miranda is $r = h + R_U$, where $h = 1.04\times10^8$ m is the altitude of Miranda above the surface of Uranus.

EXECUTE: **(a)** From the value of g at the poles,

$$m_U = \frac{g_U R_U^2}{G} = \frac{(11.1\text{ m/s}^2)(2.556\times10^7\text{ m})^2}{(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)} = 1.09\times10^{26}\text{ kg.}$$

(b) $Gm_U/r^2 = g_U(R_U/r)^2 = 0.432\text{ m/s}^2$.

(c) $Gm_M/R_M^2 = 0.080\text{ m/s}^2$.

EVALUATE: **(d)** No. Both the object and Miranda are in orbit together around Uranus, due to the gravitational force of Uranus. The object has additional force toward Miranda.

13.79. **IDENTIFY** and **SET UP:** Apply conservation of energy (Eq. (7.13)) and solve for W_{other}. Only $r = h + R_E$ is given, so use Eq. (13.10) to relate r and v.

EXECUTE: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$U_1 = -Gm_M m/r_1$, where m_M is the mass of Mars and $r_1 = R_M + h$, where R_M is the radius of Mars and $h = 2000\times10^3$ m.

$$U_1 = -(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{(6.42\times10^{23}\text{ kg})(5000\text{ kg})}{3.40\times10^6\text{ m} + 2000\times10^3\text{ m}} = -3.9667\times10^{10}\text{ J}$$

$U_2 = -Gm_M m/r_2$, where r_2 is the new orbit radius.

$$U_2 = -(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{(6.42\times10^{23}\text{ kg})(5000\text{ kg})}{3.40\times10^6\text{ m} + 4000\times10^3\text{ m}} = -2.8950\times10^{10}\text{ J}$$

For a circular orbit $v = \sqrt{Gm_M/r}$ (Eq. (13.10)), with the mass of Mars rather than the mass of the earth).

Using this gives $K = \frac{1}{2}mv^2 = \frac{1}{2}m(Gm_M/r) = \frac{1}{2}Gm_M m/r$, so $K = -\frac{1}{2}U$.

$K_1 = -\frac{1}{2}U_1 = +1.9833\times10^{10}$ J and $K_2 = -\frac{1}{2}U_2 = +1.4475\times10^{10}$ J

Then $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives

$$W_{\text{other}} = (K_2 - K_1) + (U_2 - U) = (1.4475\times10^{10}\text{ J} - 1.9833\times10^{10}\text{J}) + (+3.9667\times10^{10}\text{ J} - 2.8950\times10^{10}\text{ J})$$

$$W_{\text{other}} = -5.3580\times10^9\text{ J} + 1.0717\times10^{10}\text{ J} = 5.36\times10^9\text{ J.}$$

EVALUATE: When the orbit radius increases the kinetic energy decreases and the gravitational potential energy increases. $K = -U/2$ so $E = K + U = -U/2$ and the total energy also increases (becomes less negative). Positive work must be done to increase the total energy of the satellite.

13.80. **IDENTIFY** and **SET UP:** Use Eq. (13.17) to calculate a. $T = 30{,}000\text{ y}(3.156\times10^7\text{ s/1 y}) = 9.468\times10^{11}\text{ s}$

EXECUTE: Eq. (13.17): $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}},\ T^2 = \dfrac{4\pi^2 a^3}{Gm_S}$

$$a = \left(\frac{Gm_S T^2}{4\pi^2}\right)^{1/3} = 1.4\times10^{14}\text{ m.}$$

EVALUATE: The average orbit radius of Pluto is 5.9×10^{12} m (Appendix F); the semi-major axis for this comet is larger by a factor of 24.

$4.3\text{ light years} = 4.3\text{ light years}(9.461\times10^{15}\text{ m/1 light year}) = 4.1\times10^{16}\text{ m}$

The distance of Alpha Centauri is larger by a factor of 300.

The orbit of the comet extends well past Pluto but is well within the distance to Alpha Centauri.

13.81. **IDENTIFY:** Integrate $dm = \rho dV$ to find the mass of the planet. Outside the planet, the planet behaves like a point mass, so at the surface $g = GM/R^2$.

SET UP: A thin spherical shell with thickness dr has volume $dV = 4\pi r^2 dr$. The earth has radius $R_E = 6.38\times10^6$ m.

EXECUTE: Get $M: M = \int dm = \int \rho dV = \int \rho 4\pi r^2 dr$. The density is $\rho = \rho_0 - br$, where

$\rho_0 = 15.0\times10^3\text{ kg/m}^3$ at the center and at the surface, $\rho_S = 2.0\times10^3\text{ kg/m}^3$, so $b = \dfrac{\rho_0 - \rho_s}{R}$.

$$M = \int_0^R (\rho_0 - br)\,4\pi r^2 dr = \frac{4\pi}{3}\rho_0 R^3 - \pi b R^4 = \frac{4}{3}\pi R^3 \rho_0 - \pi R^4\left(\frac{\rho_0 - \rho_s}{R}\right) = \pi R^3\left(\frac{1}{3}\rho_0 + \rho_s\right)\text{ and}$$

$$M = 5.71\times10^{24}\text{ kg. Then } g = \frac{GM}{R^2} = \frac{G\pi R^3\left(\frac{1}{3}\rho_0 + \rho_s\right)}{R^2} = \pi RG\left(\frac{1}{3}\rho_0 + \rho_s\right).$$

$$g = \pi(6.38\times10^6\text{m})(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\left(\frac{15.0\times10^3\text{ kg/m}^3}{3} + 2.0\times10^3\text{ kg/m}^3\right).$$

$g = 9.36\text{ m/s}^2$.

EVALUATE: The average density of the planet is

$$\rho_{av} = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3} = \frac{3(5.71\times10^{24}\text{ kg})}{4\pi(6.38\times10^6\text{ m})^3} = 5.25\times10^3\text{ kg/m}^3.$$ Note that this is not $(\rho_0 + \rho_s)/2$.

13.82. **IDENTIFY** and **SET UP:** Use Eq. (13.1) to calculate the force between the point mass and a small segment of the semicircle.

EXECUTE: The radius of the semicircle is $R = L/\pi$.

Divide the semicircle up into small segments of length $R\,d\theta$, as shown in Figure 13.82.

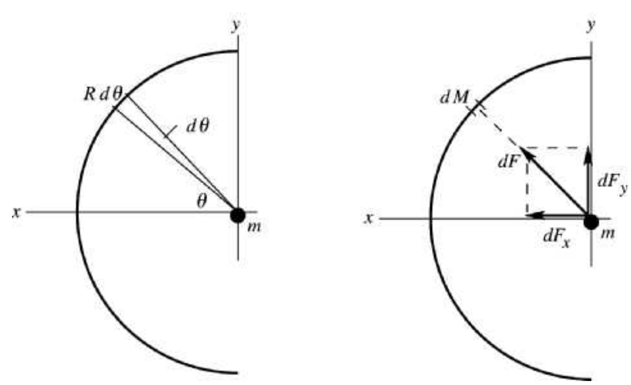


Figure 13.82

$dM = (M/L)R\,d\theta = (M/\pi)\,d\theta$

$d\vec{F}$ is the gravity force on m exerted by dM

$\int dF_y = 0;$ the y-components from the upper half of the semicircle cancel the y-components from the lower half.

The x-components are all in the $+x$-direction and all add.

$$dF = G\frac{mdM}{R^2}$$

$$dF_x = G\frac{mdM}{R^2}\cos\theta = \frac{Gm\pi M}{L^2}\cos\theta\,d\theta$$

$$F_x = \int_{-\pi/2}^{\pi/2} dF_x = \frac{Gm\pi M}{L^2}\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta = \frac{Gm\pi M}{L^2}(2)$$

$$F = \frac{2\pi GmM}{L^2}$$

EVALUATE: If the semicircle were replaced by a point mass M at $x = R$, the gravity force would be $GmM/R^2 = \pi^2 GmM/L^2$. This is $\pi/2$ times larger than the force exerted by the semicirclar wire. For the semicircle it is the x-components that add, and the sum is less than if the force magnitudes were added.

13.83. **IDENTIFY:** The direct calculation of the force that the sphere exerts on the ring is slightly more involved than the calculation of the force that the ring exerts on the sphere. These forces are equal in magnitude but opposite in direction, so it will suffice to do the latter calculation. By symmetry, the force on the sphere will be along the axis of the ring in Figure E13.33 in the textbook, toward the ring.

SET UP: Divide the ring into infinitesimal elements with mass dM.

EXECUTE: Each mass element dM of the ring exerts a force of magnitude $\dfrac{(Gm)dM}{a^2+x^2}$ on the sphere, and the x-component of this force is $\dfrac{GmdM}{a^2+x^2}\dfrac{x}{\sqrt{a^2+x^2}} = \dfrac{GmdMx}{(a^2+x^2)^{3/2}}$.

Therefore, the force on the sphere is $GmMx/(a^2+x^2)^{3/2}$, in the $-x$-direction. The sphere attracts the ring with a force of the same magnitude.

EVALUATE: As $x \gg a$ the denominator approaches x^3 and $F \to \dfrac{GMm}{x^2}$, as expected.

13.84. **IDENTIFY:** Use Eq. (13.1) for the force between a small segment of the rod and the particle. Integrate over the length of the rod to find the total force.

SET UP: Use a coordinate system with the origin at the left-hand end of the rod and the x'-axis along the rod, as shown in Figure 13.84. Divide the rod into small segments of length dx'. (Use x' for the coordinate so not to confuse with the distance x from the end of the rod to the particle.)

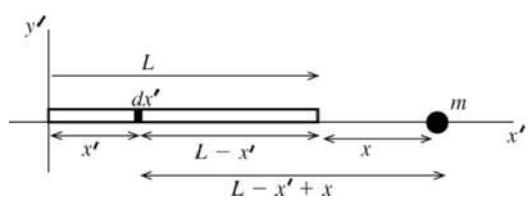


Figure 13.84

EXECUTE: The mass of each segment is $dM = dx'(M/L)$. Each segment is a distance $L - x' + x$ from mass m, so the force on the particle due to a segment is $dF = \dfrac{Gm\,dM}{(L-x'+x)^2} = \dfrac{GMm}{L}\dfrac{dx'}{(L-x'+x)^2}$.

$$F = \int_L^0 dF = \frac{GMm}{L}\int_L^0 \frac{dx'}{(L-x'+x)^2} = \frac{GMm}{L}\left(-\frac{1}{L-x'+x}\Big|_L^0\right)$$

$$F = \frac{GMm}{L}\left(\frac{1}{x} - \frac{1}{L+x}\right) = \frac{GMm}{L}\frac{(L+x-x)}{x(L+x)} = \frac{GMm}{x(L+x)}$$

EVALUATE: For $x \gg L$ this result becomes $F = GMm/x^2$, the same as for a pair of point masses.

13.85. **IDENTIFY:** Compare F_E to Hooke's law.

SET UP: The earth has mass $m_E = 5.97\times10^{24}$ kg and radius $R_E = 6.38\times10^6$ m.

EXECUTE: **(a)** For $F_x = -kx$, $U = \frac{1}{2}kx^2$. The force here is in the same form, so by analogy $U(r) = \frac{Gm_E m}{2R_E^3}r^2$. This is also given by the integral of F_g from 0 to r with respect to distance.

(b) From part (a), the initial gravitational potential energy is $\frac{Gm_E m}{2R_E}$. Equating initial potential energy and final kinetic energy (initial kinetic energy and final potential energy are both zero) gives $v^2 = \frac{Gm_E}{R_E}$, so $v = 7.90\times10^3$ m/s.

EVALUATE: When $r = 0$, $U(r) = 0$, as specified in the problem.

13.86. **IDENTIFY:** In Eqs. (13.12) and (13.16) replace T by $T + \Delta T$ and r by $r + \Delta r$. Use the expression in the hint to simplify the resulting equations.

SET UP: The earth has $m_E = 5.97\times10^{24}$ kg and $R = 6.38\times10^6$ m. $r = h + R_E$, where h is the altitude above the surface of the earth.

EXECUTE: **(a)** $T = \frac{2\pi r^{3/2}}{\sqrt{GM_E}}$ therefore

$$T + \Delta T = \frac{2\pi}{\sqrt{GM_E}}(r+\Delta r)^{3/2} = \frac{2\pi r^{3/2}}{\sqrt{GM_E}}\left(1+\frac{\Delta r}{r}\right)^{3/2} \approx \frac{2\pi r^{3/2}}{\sqrt{GM_E}}\left(1+\frac{3\Delta r}{2r}\right) = T + \frac{3\pi r^{1/2}\Delta r}{\sqrt{GM_E}}.$$

Since $v = \sqrt{\frac{GM_E}{r}}$, $\Delta T = \frac{3\pi\,\Delta r}{v}$. $v = \sqrt{GM_E}\,r^{-1/2}$, and therefore

$$v - \Delta v = \sqrt{GM_E}\,(r+\Delta r)^{-1/2} = \sqrt{GM_E}\,r^{-1/2}\left(1+\frac{\Delta r}{r}\right)^{-1/2} \text{ and } v \approx \sqrt{GM_E}\,r^{-1/2}\left(1-\frac{\Delta r}{2r}\right) = v - \frac{\sqrt{GM_E}}{2r^{3/2}}\Delta r.$$

Since $T = \frac{2\pi r^{3/2}}{\sqrt{GM_E}}$, $\Delta v = \frac{\pi\Delta r}{T}$.

(b) Starting with $T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$ (Eq. (13.12), $T = 2\pi r/v$, and $v = \sqrt{\frac{GM}{r}}$ (Eq. (13.10)), find the velocity and period of the initial orbit: $v = \sqrt{\frac{(6.673\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\,\text{kg})}{6.776\times10^6\,\text{m}}} = 7.672\times10^3$ m/s, and $T = 2\pi r/v = 5549\text{ s} = 92.5$ min. We then can use the two derived equations to approximate ΔT and Δv:

$\Delta T = \frac{3\pi\Delta r}{v} = \frac{3\pi\,(100\text{ m})}{7.672\times10^3\text{ m/s}} = 0.1228$ s and $\Delta v = \frac{\pi\Delta r}{T} = \frac{\pi(100\text{ m})}{(5549\text{ s})} = 0.05662$ m/s. Before the cable breaks, the shuttle will have traveled a distance d, $d = \sqrt{(125\text{ m}^2) - (100\text{ m}^2)} = 75$ m.

$t = (75\text{ m})/(0.05662\text{ m/s}) = 1324.7\text{ s} = 22$ min. It will take 22 minutes for the cable to break.

(c) The ISS is moving faster than the space shuttle, so the total angle it covers in an orbit must be 2π radians more than the angle that the space shuttle covers before they are once again in line. Mathematically, $\frac{vt}{r} - \frac{(v-\Delta v)t}{(r+\Delta r)} = 2\pi$. Using the binomial theorem and neglecting terms of order

$\Delta v \Delta r,\ \frac{vt}{r} - \frac{(v-\Delta v)t}{r}(1+\frac{\Delta r}{r})^{-1} \approx t\left(\frac{\Delta v}{r} + \frac{v\Delta r}{r^2}\right) = 2\pi.$ Therefore, $t = \frac{2\pi r}{\left(\Delta v + \frac{v\Delta r}{r}\right)} = \frac{v_T}{\frac{\pi\,\Delta r}{T} + \frac{v\Delta r}{r}}.$ Since

$2\pi\, r = vT$ and $\Delta r = \frac{v\Delta T}{3\pi},\ \ t = \frac{vT}{\frac{\pi}{t}\left(\frac{v\Delta T}{3\pi}\right) + \frac{2\pi}{T}\left(\frac{v\Delta T}{3\pi}\right)} = \frac{T^2}{\Delta T},$ as was to be shown.

$t = \frac{T^2}{\Delta T} = \frac{(5549\ \text{s})^2}{(0.1228\ \text{s})} = 2.5\times10^8\ \text{s} = 2900\ \text{d} = 7.9\ \text{y}.$ It is highly doubtful the shuttle crew would survive the congressional hearings if they miss!

EVALUATE: When the orbit radius increases, the orbital period increases and the orbital speed decreases.

13.87. **IDENTIFY:** Apply Eq. (13.19) to the transfer orbit.

SET UP: The orbit radius for earth is $r_\text{E} = 1.50\times10^{11}$ m and for Mars it is $r_\text{M} = 2.28\times10^{11}$ m. From Figure 13.18 in the textbook, $a = \frac{1}{2}(r_\text{E} + r_\text{M})$.

EXECUTE: **(a)** To get from the circular orbit of the earth to the transfer orbit, the spacecraft's energy must increase, and the rockets are fired in the direction opposite that of the motion, that is, in the direction that increases the speed. Once at the orbit of Mars, the energy needs to be increased again, and so the rockets need to be fired in the direction opposite that of the motion. From Figure 13.18 in the textbook, the semimajor axis of the transfer orbit is the arithmetic average of the orbit radii of the earth and Mars, and so from Eq. (13.13), the energy of the spacecraft while in the transfer orbit is intermediate between the energies of the circular orbits. Returning from Mars to the earth, the procedure is reversed, and the rockets are fired against the direction of motion.

(b) The time will be half the period as given in Eq. (13.17), with the semimajor axis equal to $a = \frac{1}{2}(r_\text{E} + r_\text{M}) = 1.89\times10^{11}$ m so

$t = \frac{T}{2} = \frac{\pi\ (1.89\times10^{11}\ \text{m})^{3/2}}{\sqrt{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})}} = 2.24\times10^7\ \text{s} = 259\ \text{days},$ which is more than $8\frac{1}{2}$ months.

(c) During this time, Mars will pass through an angle of $(360°)\frac{(2.24\times10^7\ \text{s})}{(687\ \text{d})(86{,}400\ \text{s/d})} = 135.9°,$ and the spacecraft passes through an angle of $180°,$ so the angle between the earth-sun line and the Mars-sun line must be $44.1°.$

EVALUATE: The period T for the transfer orbit is 526 days, the average of the orbital periods for earth and Mars.

13.88. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each ear.

SET UP: Denote the orbit radius as r and the distance from this radius to either ear as $\delta.$ Each ear, of mass $m,$ can be modeled as subject to two forces, the gravitational force from the black hole and the tension force (actually the force from the body tissues), denoted by F.

EXECUTE: The force equation for either ear is $\frac{GMm}{(r+\delta)^2} - F = m\omega^2(r+\delta),$ where δ can be of either sign.

Replace the product $m\omega^2$ with the value for $\delta = 0,\ \ m\omega^2 = GMm/r^3,$ and solve for F:

$$F = (GMm)\left[\frac{r+\delta}{r^3} - \frac{1}{(r+\delta)^2}\right] = \frac{GMm}{r^3}\left[r+\delta - r(1+(\delta/r)^{-2}\right].$$

Using the binomial theorem to expand the term in square brackets in powers of δ/r,

$$F \approx \frac{GMm}{r^3}[r+\delta-r(1-2(\delta/r))] = \frac{GMm}{r^3}(3\delta) = 2.1\ \text{kN}.$$

This tension is much larger than that which could be sustained by human tissue, and the astronaut is in trouble.

(b) The center of gravity is not the center of mass. The gravity force on the two ears is not the same.
EVALUATE: The tension between her ears is proportional to their separation.

13.89. **IDENTIFY:** As suggested in the problem, divide the disk into rings of radius r and thickness dr.

SET UP: Each ring has an area $dA = 2\pi r\, dr$ and mass $dM = \frac{M}{\pi a^2} dA = \frac{2M}{a^2} r\, dr.$

EXECUTE: The magnitude of the force that this small ring exerts on the mass m is then $(Gm\, dM)(x/(r^2+x^2)^{3/2})$. The contribution dF to the force is $dF = \frac{2GMmx}{a^2}\frac{rdr}{(x^2+r^2)^{3/2}}$.

The total force F is then the integral over the range of r;

$$F = \int dF = \frac{2GMmx}{a^2}\int_0^a \frac{r}{(x^2+r^2)^{3/2}} dr.$$

The integral (either by looking in a table or making the substitution $u = r^2 + a^2$) is

$$\int_0^a \frac{r}{(x^2+r^2)^{3/2}} dr = \left[\frac{1}{x} - \frac{1}{\sqrt{a^2+x^2}}\right] = \frac{1}{x}\left[1 - \frac{x}{\sqrt{a^2+x^2}}\right].$$

Substitution yields the result $F = \frac{2GMm}{a^2}\left[1 - \frac{x}{\sqrt{a^2+x^2}}\right]$. The force on m is directed toward the center of the ring. The second term in brackets can be written as

$$\frac{1}{\sqrt{1+(a/x)^2}} = (1+(a/x)^2)^{-1/2} \approx 1 - \frac{1}{2}\left(\frac{a}{x}\right)^2$$

if $x \gg a$, where the binomial expansion has been used. Substitution of this into the above form gives

$F \approx \frac{GMm}{x^2}$, as it should.

EVALUATE: As $x \to 0$, the force approaches a constant.

13.90. **IDENTIFY:** Divide the rod into infinitesimal segments. Calculate the force each segment exerts on m and integrate over the rod to find the total force.

SET UP: From symmetry, the component of the gravitational force parallel to the rod is zero. To find the perpendicular component, divide the rod into segments of length dx and mass $dm = dx\frac{M}{2L}$, positioned at a distance x from the center of the rod.

EXECUTE: The magnitude of the gravitational force from each segment is

$dF = \frac{Gm\, dM}{x^2+a^2} = \frac{GmM}{2L}\frac{dx}{x^2+a^2}$. The component of dF perpendicular to the rod is $dF\frac{a}{\sqrt{x^2+a^2}}$ and so the net gravitational force is $F = \int_{-L}^{L} dF = \frac{GmMa}{2L}\int_{-L}^{L}\frac{dx}{(x^2+a^2)^{3/2}}$.

The integral can be found in a table, or found by making the substitution $x = a\tan\theta$. Then, $dx = a\sec^2\theta\, d\theta, (x^2+a^2) = a^2\sec^2\theta,$ and so

$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \int \frac{a\sec^2\theta\, d\theta}{a^3\sec^3\theta} = \frac{1}{a^2}\int \cos\theta\, d\theta = \frac{1}{a^2}\sin\theta = \frac{x}{a^2\sqrt{x^2+a^2}},$$

and the definite integral is $F = \frac{GmM}{a\sqrt{a^2+L^2}}$.

EVALUATE: When $a \gg L$, the term in the square root approaches a^2 and $F \to \frac{GmM}{a^2}$, as expected.