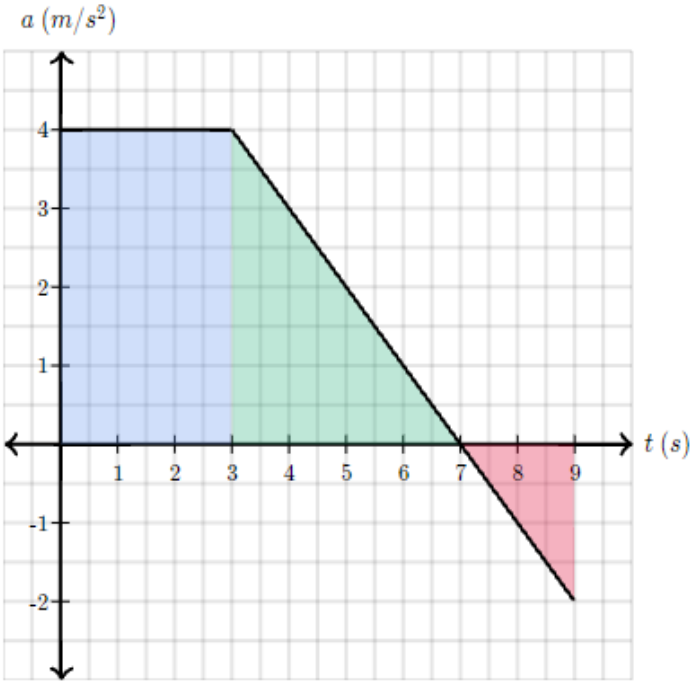
Change in Velocity from Acceleration Graph

A sailboat is sailing in a straight line with a velocity of 10 m/s. Then at time $t = 0$ s, a stiff wind blows causing the sailboat to accelerate as seen in the diagram below.

What is the velocity of the sailboat after the wind has blown for 9 seconds?

The area under the graph will give the change in velocity. The area of the graph can be broken into a rectangle, a triangle, and a triangle, as seen in the diagram below.



The blue rectangle between $t = 0$ s and $t = 3$ s is considered positive area since it is above the horizontal axis. The green triangle between $t = 3$ s and $t = 7$ s is also considered positive area since it is above the horizontal axis. The red triangle between $t = 7$ s and $t = 9$ s, however, is considered negative area since it is below the horizontal axis.

We'll add these areas together—using hw for the rectangle and $\frac{1}{2}bh$ for the triangles—to get the total area between $t = 0$ s and $t = 9$ s.

$$\Delta v = \text{area} = (4\frac{\text{m}}{\text{s}^2})(3\text{ s}) + \frac{1}{2}(4\text{ s})(4\frac{\text{m}}{\text{s}^2}) + \frac{1}{2}(2\text{ s})(-2\frac{\text{m}}{\text{s}^2})$$

(Add areas of rectangle and two triangles.)

$\Delta v = 18\text{ m/s}$ (Calculate to get total change in velocity.)

But this is the *change* in velocity, so to find the *final* velocity, we'll use the definition of change in velocity.

$v_f - v_i = 18\text{ m/s}$ (Use definition of change in velocity.)

$v_f = 18\text{ m/s} + v_i$ (Solve for the final velocity.)

$v_f = 18\text{ m/s} + 10\text{ m/s}$ (Plug in initial velocity.)

$v_f = 28\text{ m/s}$ (Calculate and celebrate!)

The final velocity of the sailboat is $v_f = 28\text{ m/s}$.