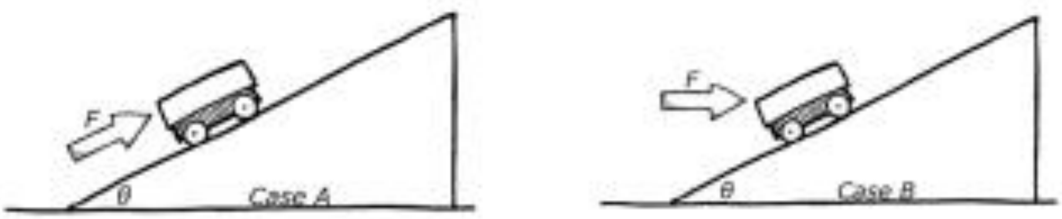
NAME ______________________________ DATE ______________

## Scenario

*Consider a cart on a smooth incline. The incline makes an angle θ with the horizontal. In two cases, the cart is pushed up the incline by a force F, whose magnitude is greater than the weight of the cart. In Case A, the force is applied parallel to the incline; in Case B, the force is applied horizontally (parallel to the ground). In both cases, the cart is pushed from rest a length L up the incline and attains a final speed $v_A$ in Case A and a final speed $v_B$ in Case B.*




## Argumentation

PART A: The two final speeds are related according to the inequality $v_A > v_B$.

i. Explain why this is the case by discussing specific forces and force components.

Using Newton's second law, $F_{net} = ma$, we can see that the net force in Case A will be larger than in Case B. This is because in Case A, the force pushing up the ramp is the entire magnitude of F, while the force exerted down the ramp is the parallel component of the block's weight. In Case B, only a component of the force (so less than in Case A) pushes up the ramp, while the same parallel component of weight is exerted down the ramp. From here, we can conclude that the acceleration in Case A will be larger, and given the kinematics equation $v_f^2 = v_i^2 + 2aL$, we can see that the final velocity in Case A will be larger as well. It should be noted L and $v_i$ are the same value for both cases.

ii. Explain why this is the case by discussing specific energy transformations.

Using the fact that the cart can be modeled as an object, $W_{net} = \Delta K$ and that $W_{net} = F_{net}d$, we can again argue from the above statements that the $F_{net}$ in Case A is larger while the $d$ is just L, so it's the same value for each case concluding that the work in Case A will be more. Thus, we know the $\Delta K$ in Case A is more and $\Delta K = \frac{1}{2}m(v_f^2 - v_i^2)$ where again $v_i$ is the same for each case and the mass of the block is the same, so the final velocity must be more in Case A.

## Quantitative Analysis

**PART B:** Using either Newton's second law and a kinematics equation or using principles of work and energy, write equations for the following in terms of $m$, $g$, $L$, $\theta$, and $F$.

i. The speed $v_A$

Start with the key principle: $W_{net} = \Delta K$

Expand each term: $F_{net}d = \frac{1}{2}m(v_f^2 - v_i^2)$

Write out the parallel component of weight using $W = mg$ and trigonometry: $(F - mg\sin\theta)L = \frac{1}{2}m(v_f^2 - v_i^2)$.

Solve for $v_f$ from the above expression, and $v_i = 0$:

$$v_{fA} = \sqrt{\frac{2L(F - mg\sin\theta)}{m}}$$

Check that the answer has proper units: $F/m$ and $g\sin\theta$ have units of $m/s^2$. Multiplying those by L gives us $m^2/s^2$ which we then square to leave m/s, which is the proper dimension for velocity.

ii. The speed $v_B$

Start with the key principle: $W_{net} = \Delta K$.

Expand each term: $F_{net}d = \frac{1}{2}m(v_f^2 - v_i^2)$.

Write out the parallel component of weight using $W = mg$ and trigonometry: $(F\cos\theta - mg\sin\theta)L = \frac{1}{2}m(v_f^2 - v_i^2)$.

Solve for $v_f$ from the above expression and $v_i = 0$:

$$v_{fB} = \sqrt{\frac{2L(F\cos\theta - mg\sin\theta)}{m}}$$

Check that the answer has proper units: $\frac{F\cos\theta}{m}$ and $g\sin\theta$ have units of $m/s^2$. Multiplying those by L gives us $m^2/s^2$, which we then square to leave m/s, which is the proper dimension for velocity.

**PART C:** Explain how your two equations from Part B support the idea that $v_A > v_B$. Also explain how your two equations support your answer to either Part A (i) or Part A (ii), depending on whether you used forces or energy in Part B.

Here are the two speeds we derived above:

$v_{fA} = \sqrt{\dfrac{2L(F - mg\sin\theta)}{m}}$ $\quad v_{fB} = \sqrt{\dfrac{2L(F\cos\theta - mg\sin\theta)}{m}}$

Given that $\cos\theta$ is less than one in case B, we can conclude mathematically that $v_A$ must be larger as it starts with a larger number and then subtracts off the same term as in Case B. This agrees with BOTH Part A (i) and (ii) arguments because in each argument, it was stated that the net force for Case B will be less because the upward push is only a fraction of the magnitude F.